Tutorial 2, Jan 14, 2026

3-Wheel Omnidirectional Robot Example

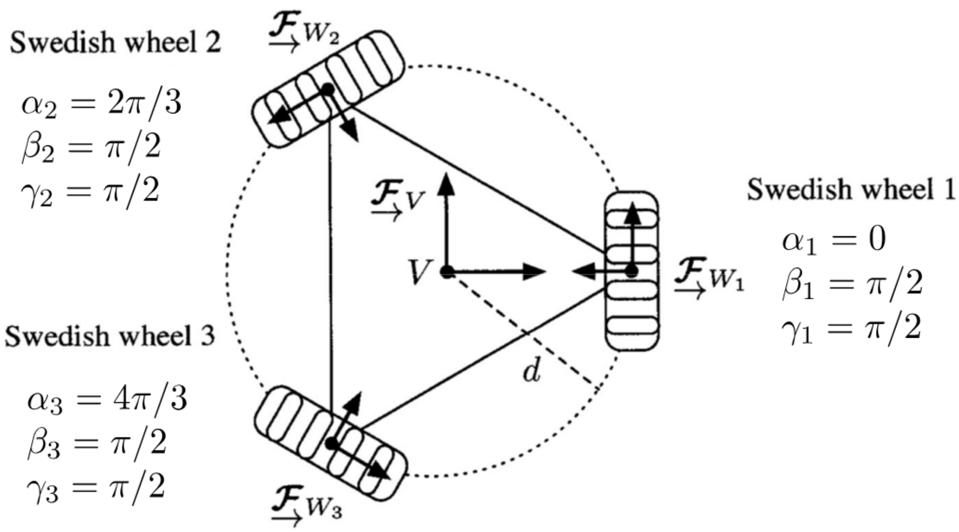

Figure 1: Setup of the robot.

- Consider a 3-wheeled omnidirectional robot, with wheels arranged in the diagram and rollers arranged so that the wheel can roll laterally
- Recall the set of Swedish wheel constraints:
  - $\begin{bmatrix}\cos(\alpha+\beta+\gamma) & \sin(\alpha+\beta+\gamma) & d\sin(\beta+\gamma)\end{bmatrix}\dot{\boldsymbol{\xi}} = \dot{\varphi} r\cos\gamma + \dot{\varphi}_s r_s$
  - $\begin{bmatrix}-\sin(\alpha+\beta+\gamma) & \cos(\alpha+\beta+\gamma) & d\cos(\beta+\gamma)\end{bmatrix}\dot{\boldsymbol{\xi}} = -\dot{\varphi} r\sin\gamma$
- Substituting $\gamma = \pi/2$ for all 3 wheels, and ignoring the first constraint since the rollers are free:
  - $$\begin{bmatrix}\cos\pi/2 & \sin\pi/2 & d\sin\pi/2\\ \cos 7\pi/6 & \sin 7\pi/6 & d\sin\pi/2\\ \cos 11\pi/6 & \sin 11\pi/6 & d\sin\pi/2\end{bmatrix}\dot{\boldsymbol{\xi}} = r\begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\\ \dot{\varphi}_3\end{bmatrix}$$
  $$\implies \begin{bmatrix}0 & 1 & d\\ -\sqrt{3}/2 & -1/2 & d\\ \sqrt{3}/2 & -1/2 & d\end{bmatrix}\begin{bmatrix}v\\ u\\ \omega\end{bmatrix} = r\begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\\ \dot{\varphi}_3\end{bmatrix}$$
- This gives the inverse differential kinematics: $\begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\\ \dot{\varphi}_3\end{bmatrix} = \frac{1}{r}\begin{bmatrix}0 & 1 & d\\ -\sqrt{3}/2 & -1/2 & d\\ \sqrt{3}/2 & -1/2 & d\end{bmatrix}\begin{bmatrix}v\\ u\\ \omega\end{bmatrix} = \boldsymbol{A}\dot{\boldsymbol{\xi}}$
- To get the forward kinematics we multiply by $\boldsymbol{A}^{-1}$ to get $\begin{bmatrix}v\\ u\\ \omega\end{bmatrix} = r\begin{bmatrix}0 & -1/\sqrt{3} & 1/\sqrt{3}\\ 2/3 & -1/3 & -1/3\\ 1/(3d) & 1/(3d) & 1/(3d)\end{bmatrix}\begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\\ \dot{\varphi}_3\end{bmatrix}$
- What if we switch the first wheel to a standard wheel?
  - We need to add the standard wheel lateral constraint $\begin{bmatrix}-\sin\pi/2 & \cos\pi/2 & d\cos\pi/2\end{bmatrix}\dot{\boldsymbol{\xi}} = 0$
  - This results in the new kinematics $\begin{bmatrix}0 & 1 & d\\ -\sqrt{3}/2 & -1/2 & d\\ \sqrt{3}/2 & -1/2 & d\\ -1 & 0 & 0\end{bmatrix}\begin{bmatrix}v\\ u\\ \omega\end{bmatrix} = r\begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\\ \dot{\varphi}_3\\ 0\end{bmatrix}$
  - Notice now that we are forcing v to be zero, but we can still freely control u and ω
  - We can also simply the model down to completely eliminate v now since the vehicle only have 2 degrees of freedom