Tutorial 4, Feb 2, 2026

- PBH test proof: (A, B) is controllable if and only if each $\lambda \in \sigma(A)$ is controllable
  - Forward: suppose (A, B) is controllable, and for contradiction suppose some λ is not controllable, so rank $\begin{bmatrix} A - \lambda I & B \end{bmatrix} < n$
    * The rows of the matrix are dependent, therefore $\exists v \in \mathbb{R}^n, v \neq 0$ such that $v^T \begin{bmatrix} A - \lambda I & B \end{bmatrix} = 0$
    * Therefore $v^T(A - \lambda I) = v^T B = 0 \implies v^T A = \lambda v^T$
    * $v^T AB = \lambda v^T B = 0$, $v^T A^2 B = \lambda^2 v^T B = 0$ and so on
    * Therefore $v^T Q_c = v^T \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix} = 0$, so Q_c is not full row rank, therefore (A, B) is not controllable