Tutorial 10, Mar 23, 2026

- To model the exosystem we often need to discretize a continuous time signal by sampling, since in real life the external signals are usually continuous time
  - Example: $r(t) = u(t) \to r(k) = 1$
    - $r(k+1) = r(k) = 1$
    - Therefore we let $w(k) = r(k), w(0) = 1, w(k+1) = w(k)$
  - Example: $r(t) = t \to r(k) = kT$
    - $r(k+1) = kT + T = r(k) + T$
    - $r(k+2) = kT + 2T = r(k+1) + T = r(k+1) + (r(k+1) - r(k)) = 2r(k+1) - r(k)$
    - Let $w(k) = \begin{bmatrix} r(k) \\ r(k+1) \end{bmatrix} \implies w(k+1) = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} w(k), r(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} w(k)$
    - $w(0) = \begin{bmatrix} r(0) \\ r(1) \end{bmatrix} = \begin{bmatrix} 0 \\ T \end{bmatrix}$
  - Example: $r(t) = \alpha \sin(\omega t) + \beta \cos(\omega t) \to r(k) = \alpha \sin(\omega T k) + \beta \cos(\omega T k)$
    - $r(k+1) = \alpha \sin(\omega T k + \omega T) + \beta \cos(\omega T K + \omega T)$
      $= \alpha \sin(\omega T k) \cos(\omega T) + \alpha \cos(\omega T k) \sin(\omega T) + \beta \cos(\omega T k) \cos(\omega T) - \beta \sin(\omega T k) \sin(\omega T)$
    - Let $w(k) = \begin{bmatrix} \sin(\omega T k) \\ \cos(\omega T k) \end{bmatrix} \implies w(k+1) = \begin{bmatrix} \cos(\omega T) & \sin(\omega T) \\ -\sin(\omega T) & \cos(\omega T) \end{bmatrix} w(k), r(k) = \begin{bmatrix} \alpha & \beta \end{bmatrix} w(k)$
  - To generate $r(t) = 4 - \sin(t) + \cos(t)$, we can have $w(k) = \begin{bmatrix} 4 \\ \sin(Tk) \\ \cos(Tk) \end{bmatrix}, w(k+1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(T) & \sin(T) \\ 0 & -\sin(T) & \cos(T) \end{bmatrix} w(k)$ with $w(0) = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$