Lecture 31, Mar 26, 2026

Kreisselmeier Filters

- To construct an estimate of the effective disturbance, we can use a *Kreisselmeier filter*
- $\eta_0(k+1) = F\eta_0(k) + FGe(k)$
  $\eta_1(k+1) = F\eta_1(k) - Ge(k)$
  $\eta_2(k+1) = F\eta_2(k) + Gu(k)$
  - (F, G) is controllable and F is Schur stable, as we have used for the Nikiforov canonical forms
  - In this case η_0, η_1, η_2 are scalars, but they generalize to higher dimensions as well
- Consider the estimate: $\hat{w}(k) = \eta_0(k) + Ge(k) - A\eta_1(k) + B\eta_2(k)$
  - $$\begin{aligned}\hat{w}(k+1) &= \eta_0(k+1) + Ge(k+1) - A\eta_1(k+1) + B\eta_2(k+1) \\ &= F\eta_0(k) + FGe(k) + G(Ae(k) - Bu(k) + d_*) - A(F\eta_1(x) - Ge(k)) + B(F\eta_2(k) + Gu(k)) \\ &= F(\eta_0(k) + Ge(k) + A\eta_1(k) + B\eta_2(k)) + Gd_* \\ &= F\hat{w}(k) + Gd_*\end{aligned}$$
  - From the Nikiforov canonical representation, we know that a model for d_* is $w(k+1) = Fw(k) + Gd_*, d_* = \psi^T w(k)$
  - Now the estimation error $\tilde{w}(k+1) = \hat{w}(k+1) - w(k+1) = F\tilde{w}(k)$, and since F is Schur stable our estimate converges to $w(k)$