Lecture 30, Mar 24, 2026

Disturbance Observers

- How can we estimate $w(k)$?
- Recall that we have the system $z(k+1) = Az(k) + Bu(k) - B\psi^T w(k)$
$$e(k) = Cz(k)$$
- Consider the system: $\hat{z}_d(k+1) = A\hat{z}_d(k) + Bu(k) + L_d(e(k) - \hat{e}(k))$
$$\hat{e}(k) = C\hat{z}_d(k)$$
  - This looks like an observer, but note that we are not estimating $z(k)$ directly since we don't have the $B\psi^T w(k)$ term; instead, we want to find the missing term
  - As usual, we select L_d such that $A - L_d C$ is Schur stable
    * Note that if A is already stable then we can just choose $L_d = 0$, but using the observer gain we can make it converge faster
- Let the estimation error $\tilde{z}_d(k) = \hat{z}_d(k) - z(k)$, then $\tilde{z}_d(k+1) = (A - L_d C)\tilde{z}_d(k) + Bd(k)$
  - Let $d_f(k) = C\tilde{z}_d(k)$
$$= C\hat{z}_d(k) - Cz(k)$$
$$= C\hat{z}_d(k) - e(k)$$
    * This means $d_f(k)$ is fully measurable since we compute $\hat{z}_d(k)$ and we can measure $e(k)$
  - Note that $d_f(k)$ is just a version of $d(k)$ filtered through a stable LTI system; therefore it should have the same frequency content as $d(k)$, since LTI systems don't change frequency
  - Therefore $d_f(k)$ is generated by $w_f(k+1) = Fw_f(k) + Gd_f(k)$
$$d_f(k) = \psi^T w_f(k)$$
    * The F, G matrices and parameter vector are the same as the ones previously used for the Nikiforov representation
    * Moreover there is a coordinate transformation $w_f(k) = H_f w(k)$ where H_f is nonsingular
    * See lemma in Lecture 32 and the discussion that follows about why this is the case
- Now we can make a filter $\hat{w}_f(k+1) = F\hat{w}_f(k) + Gd_f(k)$
  - Consider the estimation error $\tilde{w}_f(k) = \hat{w}_f(k) - w_f(k)$
  - $\tilde{w}_f(k+1) = \hat{w}_f(k+1) - w_f(k+1)$
$$= F\hat{w}_f(k) + Gd_f(k) - (Fw_f(k) + Gd_f(k))$$
$$= F\tilde{w}_f(k)$$
  - Since F is Schur stable, $\hat{w}_f(k) \to w_f(k)$ exponentially
- Finally we have $d(k) = \psi^T w(k)$
$$= \psi^T H_f^{-1} w_f(k)$$
$$= \psi_f^T(\hat{w}_f(k) - \tilde{w}_f(k))$$
$$= \psi_f^T \hat{w}_f(k) + \varepsilon(k) \qquad \text{where } \varepsilon(k) \to 0$$