Lecture 3, Jan 9, 2026

Time Response

- Given a state space model $x(k+1) = Ax(k) + Bu(k), y(k) = Cx(k) + Du(k)$, an initial condition $x(0)$ and input at all times $\{ u(k) \}_{k \geq 0}$, we want to obtain an explicit formula for $x(k), y(k)$, for $k \geq 0$
- As with continuous systems, since the system is LTI we can again break the total response into a superposition of the initial state response (nonzero initial conditions with zero input) and input response (zero initial conditions with nonzero input)
  - Initial state response: $x(k+1) = Ax(k) \implies x(k) = A^k x(0)$
    * This is an explicit formula because A^k can always be computed non-iteratively as we will see later
    * A^k is the discrete time analogue of e^{At} in continuous time
  - Input response:
    $$\begin{aligned} & x(k+1) = Ax(k) + Bu(k), x(0) = 0 \\ \implies & x(k) = Bu(k-1) + ABu(k-2) + A^2Bu(k-3) + \cdots + A^{k-1}Bu(0) \\ & = \sum_{i=0}^{k-1} A^{k-1-i}Bu(i) \end{aligned}$$
    * This is the discrete time analogue of a convolution
- The total response is $x(k) = A^k x(0) + \sum_{i=0}^{k-1} A^{k-1-i}Bu(i)$