Lecture 28, Mar 19, 2026

The Regulator Problem – Partial Measurement

- If we don't have full measurement (only $e(k)$ is measureable), we need to build an observer for $\begin{bmatrix} x(k) \\ w(k) \end{bmatrix}$, so we can use our earlier design
- Consider the composite system:
$$\begin{aligned} \begin{bmatrix} x(k+1) \\ w(k+1) \end{bmatrix} &= \begin{bmatrix} A & E \\ 0 & S \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k) &&= A_c \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} + B_c u(k) \\ e(k) &= \begin{bmatrix} C & D \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} &&= C_c \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} \end{aligned}$$
- We need the following assumptions:
  1. (A, B) is controllable
  2. (C, A) is observable
  3. $\left(\begin{bmatrix} C & D \end{bmatrix}, \begin{bmatrix} A & E \\ 0 & S \end{bmatrix} \right)$ is observable
     - Note that we can assume this without loss of generality because if the composite system is unobservable, this means we have exosystem states which don't impact the error at all; we can then use a Kalman decomposition to eliminate the redundant states to make the overall system observable
- Now we have the observer:
$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + Bu(k) + E\hat{w}(k) + L_1(e(k) - \hat{e}(k)) \\ \hat{w}(k+1) &= S\hat{w}(k) + L_2(e(k) - \hat{e}(k)) \\ \hat{e}(k) &= C\hat{x}(k) + D\hat{w}(k) \end{aligned}$$
  - This observer is our internal model
- Let the estimation errors $\tilde{x}(k) = \hat{x}(k) - x(k), \tilde{w}(k) = \hat{w}(k) - w(k)$
  - The dynamics are
$$\begin{aligned} \begin{bmatrix} \tilde{x}(k+1) \\ \tilde{w}(k+1) \end{bmatrix} &= \left(\begin{bmatrix} A & E \\ 0 & S \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix} \right) \begin{bmatrix} \tilde{x}(k) \\ \tilde{w}(k) \end{bmatrix} \\ &= (A_c - LC_c) \begin{bmatrix} \tilde{x}(k) \\ \tilde{w}(k) \end{bmatrix} \end{aligned}$$
- The overall system block diagram is depicted below: