Lecture 26, Mar 13, 2026

The Regulator Problem

Definition

Regulator Problem: Given a model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ew(k) \\ w(k+1) &= Sw(k) \\ e(k) &= Cx(k) + Dw(k) \end{aligned}$$

where $x(k) \in \mathbb{R}^n, u(k) \in \mathbb{R}^m, e(k) \in \mathbb{R}^p, w(k) \in \mathbb{R}^q$, we want to design a controller (*regulator*) $u(k)$ to satisfy 2 requirements:

1. The unforced closed-loop system (i.e. $w(k) \equiv 0, \forall k \geq 0$) has an equilibrium that is asymptotically stable.
2. When $w(0) \neq 0$, $\lim_{k \to \infty} e(k) = 0$.

- Note that we assume knowledge of all the matrices in this system
  - We don't know the exact value of $w(k)$ but we know E, which reflects the fact that we know how disturbances may enter the system, but not exactly how much the disturbances are
- $w(k+1) = Sw(k)$ is the *exosystem*, and models possible disturbances to the system; we cannot measure $w(k)$ directly
- $Dw(k)$ can be seen as a reference signal
  - Note that $w(k)$ contains both the modes that correspond to possible disturbances and our reference signal, and by picking E and D we can pick them out
  - This allows us to have a single exosystem model for both
- The exosystem will be designed based on the expected reference and disturbance signals, which are usually a combination of constant signals and sinusoids