Lecture 13, Feb 3, 2026

Theoretical Justification of Error Models

Definition

The l_∞-*norm* of a discrete signal $x : \mathbb{N}_0 \to \mathbb{R}^n$ is

$$\|x\|_{l_\infty} = \sup_{k \geq 0} \|x(k)\|$$

We denote $x(k) \in l_\infty$ if $\|x\|_{l_\infty}$ exists (i.e. is finite).

Definition

The l_2-*norm* of a discrete signal $x : \mathbb{N}_0 \to \mathbb{R}^n$ is

$$\|x\|_{l_2} = \sqrt{\sum_{k=0}^{\infty} \|x(k)\|^2}$$

We denote $x(k) \in l_2$ if $\|x\|_{l_2}$ exists (i.e. is finite).

- The intuition is that l_∞ means that the signal is bounded, while l_2 means that the signal has finite energy
  - e.g. the signal $x(k) = 1$ is $x \in l_\infty$ but $x \notin l_2$; the signal $x(k) = a^k, |a| < 1$ is $x \in l_\infty$ and $x \in l_2$ since this infinite sum converges
  - l_2 signals must be decaying for the sum to converge