Tutorial 1, Sep 15, 2025

- 3 approaches to control:
  - State space – pioneered by Kalman
  - Transfer function – classical approach
  - Behavioural – data-driven approach (instead of explicit modelling)
- Consider a standard RLC circuit
  - From KVL: $u - Ri - L\frac{\mathrm{d}i}{\mathrm{d}t} - v = 0$
  - From KCL: $i = C\frac{\mathrm{d}v}{\mathrm{d}t}$
  - Input u is the voltage across a voltage source
  - Method 1: $0 = u - RC\frac{\mathrm{d}v}{\mathrm{d}t} - L\frac{\mathrm{d}}{\mathrm{d}t}\left(C\frac{\mathrm{d}v}{\mathrm{d}t}\right) - v$
    * $-LC\frac{\mathrm{d}^2v}{\mathrm{d}t^2} - RC\frac{\mathrm{d}v}{\mathrm{d}t} - v + u = 0$
    * This is the *input-output* form of the system
    * Let $\boldsymbol{x} = \begin{bmatrix} v \\ \frac{\mathrm{d}v}{\mathrm{d}t} \end{bmatrix}$
    * $\dot{\boldsymbol{x}} = \begin{bmatrix} \frac{\mathrm{d}v}{\mathrm{d}t} \\ \frac{\mathrm{d}^2v}{\mathrm{d}t^2} \end{bmatrix} = \begin{bmatrix} \frac{\mathrm{d}v}{\mathrm{d}t} \\ \frac{1}{LC}(-RC\frac{\mathrm{d}x}{\mathrm{d}t} - x + u) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix}u = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u$
    * Now we construct the output equation since we cannot directly measure all components of $\boldsymbol{x}$
    * $y = v = \begin{bmatrix} 1 & 0 \end{bmatrix}\boldsymbol{x} + 0u = \boldsymbol{C}x + Du$
  - Method 2: $\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -L\frac{\mathrm{d}i}{\mathrm{d}t} - Ri - v + u \\ -C\frac{\mathrm{d}i}{\mathrm{d}t} + i \end{bmatrix}$
    * $0 = \begin{bmatrix} -L & 0 \\ 0 & -C \end{bmatrix}\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} -R & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u$
    * This is similar to the input-output form we had before, but in a vector form
  - Generally the number of state variables we need is the number of derivatives times the number of variables
    * In the first method we used a second derivative, and only v as output
    * In the second method we used only first order derivatives but both v and i as output