Lecture 9, Sep 22, 2025

DH Tables

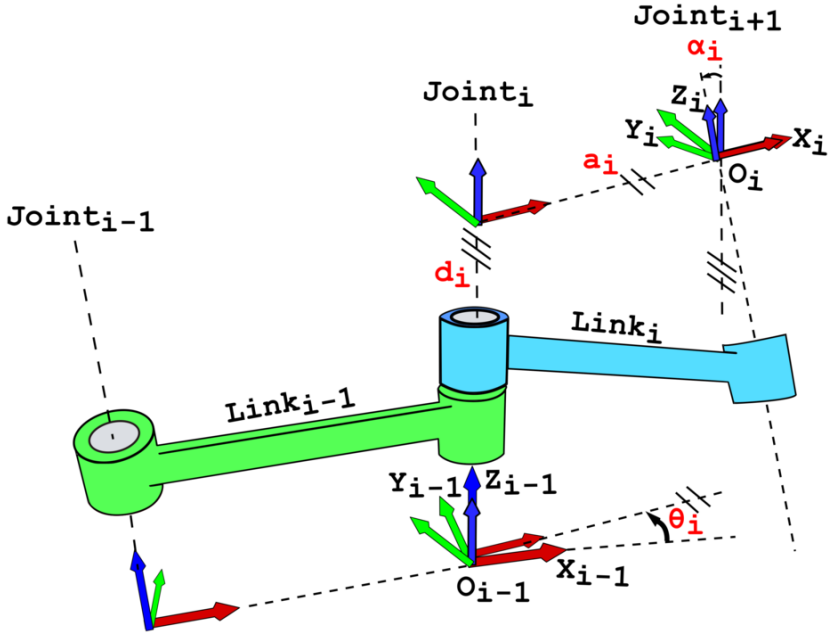

Figure 1: Illustration of the 4 DH parameters.

- Now that we have systematic frame assignments, we need a way to also systematically compute the homogeneous transformation for each joint/link
- After the frame assignment, we can describe every joint/link with just 4 parameters:
  1. Link twist α_i: the signed angle between z_{i-1} and z_i, about x_i
  2. Link length a_i: the signed distance between z_{i-1} and z_i, along x_i
  3. Link offset d_i: the signed distance between O_{i-1} and O_i, along z_{i-1}
  4. Joint angle θ_i: the signed angle between x_{i-1} and x_i, about z_{i-1}
- The angles can be better illustrated if we bring frame $i-1$ and frame i together
- This allows us to form a *DH table*, which lists out $a_i, \alpha_i, d_i, \theta_i$ for each $i \in [1, n]$
- Notice that each of the parameters corresponds to a single operation about a single axis, so we can get the overall homogeneous transformation for each stage of the manipulator by combining the 4 operations
  - $H_i^{i-1} = \text{Rot}_{z,\theta_i}\text{Trans}_{z,d_i}\text{Trans}_{x,a_i}\text{Rot}_{x,\alpha_i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$
  - The order of transformations here can be deduced by noticing the axis that each operation operates on
- Consider the example in the image; the DH table for this example is the following

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*

- Often we mark the variables that will be changed by joint movement with $*$ (these variables later become the joint variables q); the rest of the variables are rigid

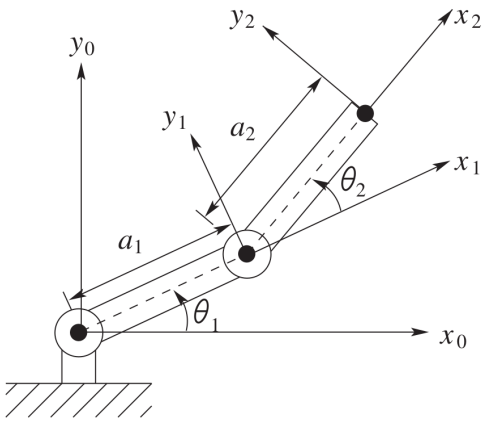

Figure 2: Example 2-stage manipulator annotated with DH parameters. For all frames, z points out of the page.

– For prismatic joints, this is always d_i, while for revolute joints this is θ_i

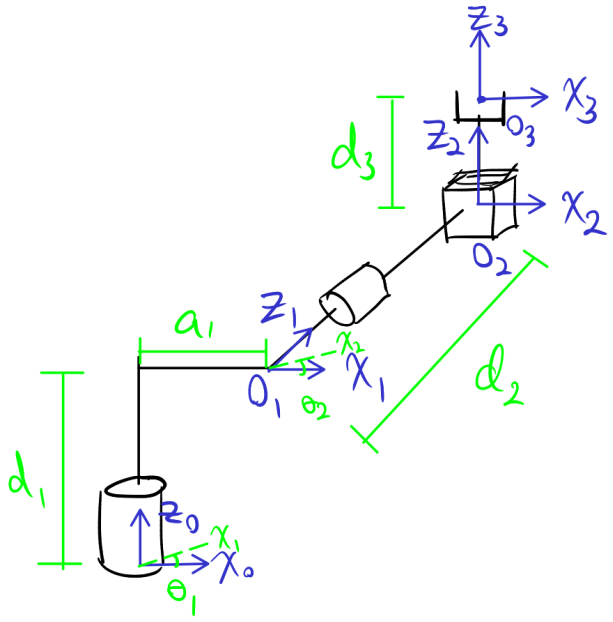

Figure 3: Another example of assigning DH parameters.

- Consider another example with an elbow in the first stage as in the above image; the DH table is the following

Link	a_i	α_i	d_i	θ_i
1	a_1	$-\pi/2$	d_1	θ_1^*
2	0	$\pi/2$	d_2	θ_2^*
3	0	0	d_3^*	0