Lecture 7, Sep 17, 2025

Denavit-Hartenberg (DH) Parameters

- Recall that we can compute the pose of the end-effector by multiplying together all the H_i^{i-1} corresponding to the homogeneous transformations of each of the joints; however, the assignment of frames is still arbitrary, and we still need a way to efficiently compute R_i^{i-1}, O_i^{i-1}
- This method of systematically assigning coordinate frames and describing the links/joints is called the *Denavit-Hartenberg* (DH) convention
- First, we assign coordinate frames according to the following rules, starting from the inertial frame 0 all the way to the end effector n:
  - Attach frames as follows:
    * Attach frame 0 (the inertial frame) to the base link, which never moves
    * Attach frame i to link i where $i \in [1, n-1]$ such that when joint i is actuated, frame i and link i move
    * Attach frame n to the end effector
  - All frames follow the two fundamental rules:
    * (DH1) x_i is orthogonal to z_{i-1}
    * (DH2) x_i intersects z_{i-1} (i.e. if we extend the axes out infinitely in both directions, there is a point where they meet)
  - Note frame n is not as constrained, so we generally put it on the end-effector

Note

Due to the Denavit-Hartenberg frame rules, the origins of the frames are not necessarily physically on the joints or links, but they are always rigid with respect to the link. For most conventional geometries however, frame i is usually at the center of joint i.

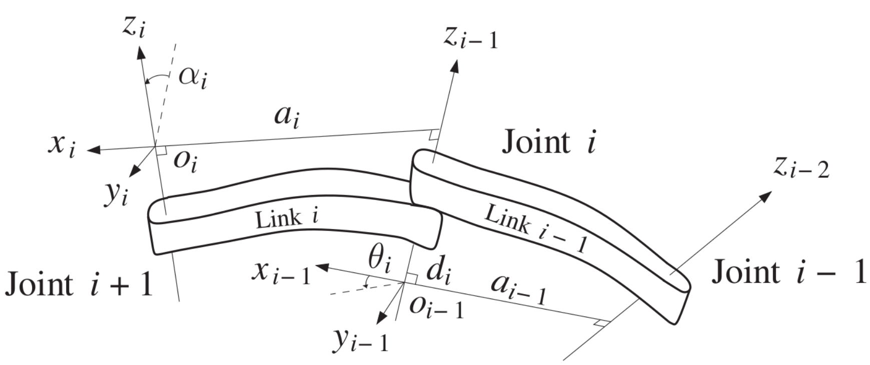

Figure 1: Assignment of DH parameters in the case where z_i and z_{i-1} are not coplanar.

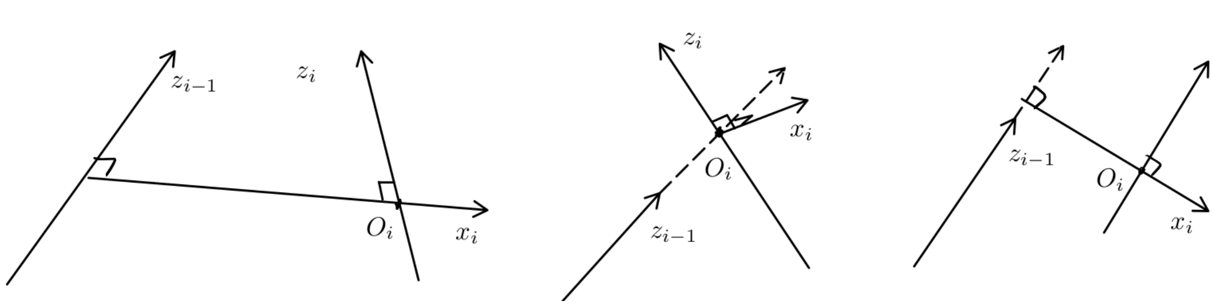

Figure 2: Illustration of the 3 cases of assigning the x axes.

- The procedure for assigning frames in detail is:

1. Assign $z_0, \dots, z_{n-1}$ such that z_i is the axis of actuation of joint $i+1$ (i.e. the axis of rotation for revolute joints, or the axis of translation for prismatic joints)
2. Choose the base frame such that we have a right-handed frame
3. Assign the x axes for each of the frames in sequence:
   - If z_{i-1} and z_i are not coplanar, then find the unique shortest line segment s between z_{i-1} and z_i to define x_i, and define O_i as the intersection of s and z_i
     * Due to the geometry, this is guaranteed to be orthogonal to both z_{i-1} and z_i, and by construction it intersects z_{i-1} and z_i
     * The point of intersection of x_i and z_i is defined as O_i
     * Choose x_i so that it is parallel to s, and in the direction towards the next link
   - If z_{i-1} and z_i intersect transversally (i.e. intersect but not parallel), declare O_i as the intersection of the vectors, and define x_i so that it is normal to the plane formed by z_{i-1} and z_i
     * Note there are two possible directions we can choose x_i; the choice does not matter
   - If z_{i-1} and z_i are parallel (or they are the same), choose O_i anywhere along z_i; x_i can be chosen as any vector orthogonal to both z_{i-1} and z_i as long as it intersects z_{i-1}
4. Assign the frame for the end-effector so that O_n is on the end-effector and select x_n to satisfy the two DH rules
   - The assignment of z_n doesn't really matter in this case (as long as it is orthogonal to x_n) so do x_n first
   - Typically the math works out easier if we make z_n parallel to z_{n-1}, if possible, so the transformation between the last 2 frames is simpler