Lecture 2, Sep 5, 2025

Robot Manipulators Basics

- Manipulators are represented by a collection of rigid links connected by joints, which can be revolute (R, rotation around an axis) or prismatic (P, translation along an axis)
  - A series of links and joints forms a *kinematic chain*
- A spherical wrist is composed of 3 revolute joints (this is the case on the lab KUKA robots)
  - Typically all 3 degrees of freedom are consolidated into a single mechanical part, but expanded out in mathematical diagrams

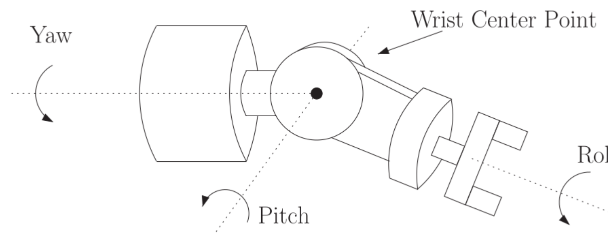

Figure 1: Spherical wrist diagram.

- The *topology* of a manipulator describes how links are connected; examples include serial chain (no loops), closed chain, or tree structure
  - Serial chains are much easier to control since we have a single fixed base, so we do not need to account for complex coupling like in closed chains
  - This course focuses on serial chains, the simplest topology
- Some popular serial chain robots:
  - Articulated manipulator: RRR, typically with wrist for 6 degrees of freedom
  - Spherical manipulator: RRP (named so because it maps out spherical coordinates)
  - SCARA: RRP, but unlike the spherical manipulator all 3 axes are parallel; typically used for pick-and-place tasks
- Our objective is to represent the pose of all the links (which leads to the pose of the end effector) as a function of the joint variables (rotation and translation)

Notation

- We fix the world coordinate frame 0, with origin (*base point*) O_0 and right-hand coordinate axes (orthonormal basis) x_0, y_0, y_0
  - Frame 0 is typically attached at base of the robot
- A superscript is used to denote the reference frame: p^0 denotes a point p in frame 0 and v^0 denotes a vector
  - Note while points are defined as an offset with respect to a reference origin, vectors are just a direction and magnitude so they do not need a coordinate origin
  - $p^0 = O_0^0 + \begin{bmatrix} a \\ b \\ c \end{bmatrix} = O_0 + ax_0^0 + by_0^0 + cz_0^0$
  - $v^0 = \begin{bmatrix} d \\ e \\ f \end{bmatrix} = dx_0^0 + ey_0^0 + fz_0^0$
  - e.g. x_0^1 is the x unit vector for frame 0, expressed in frame 1
- Note by definition $O_i^i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, x_i^i = e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, y_i^i = e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, z_i^i = e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ for any frame i
- To express p^0 in frame 1, we can write $p^1 = O_0^1 + ax_0^1 + by_0^1 + cz_0^1$ and once we find the basis vector representations, the point can be computed