Lecture 11, Sep 26, 2025

Inverse Kinematics by Kinematic Decoupling

- Recall that we decomposed the problem so that $O_c(q_1, q_2, q_3) = O_d^0 - R_d^0 \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix}$
- For inverse position kinematics, we usually do this by analyzing the geometry
- For inverse orientation kinematics, notice that the 3 joints in a spherical wrist form zyz Euler angles, so we can directly compute the joint angles (ϕ, θ, ψ) using the formula introduced previously

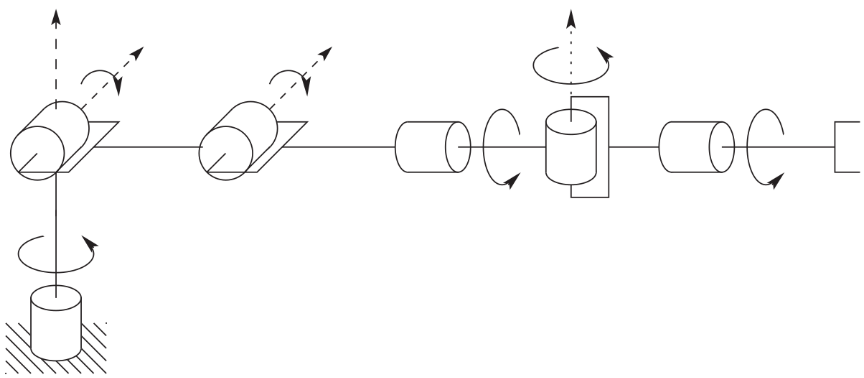

Figure 1: Elbow manipulator with spherical wrist.

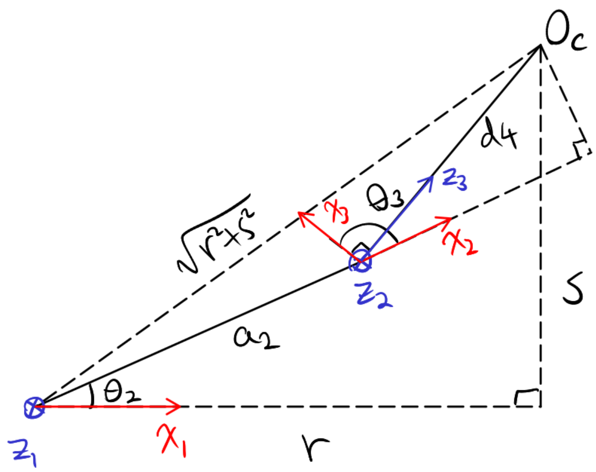

Figure 2: Diagram of link 2 and link 3 for the geometric determination of θ_2, θ_3.

- Example: Consider the RRR manipulator with spherical wrist as in the figure above
  - The center of the spherical wrist is $O_c^0 = \begin{bmatrix} x_c & y_c & z_c \end{bmatrix}^T$
  - Let $r = \sqrt{x_c^2 + y_c^2}$ be the horizontal distance from the spherical wrist center to O_0
  - Let $s = |z_c - d_1|$ be the vertical distance between the spherical wrist center and the top of link 1 (position of joint 2)
  - By geometry, $\theta_1 = \text{atan2}(y_c, x_c)$
  - Using the cosine law: $r^2 + s^2 = a_2^2 + d_4^2 - 2a_2 d_4 \cos\left(\frac{3\pi}{2} - \theta_3\right) = a_2^2 + d_4^2 + 2a_2 d_4 \sin\theta_3$
    * $\theta_3 = \sin^{-1}\left(\frac{r^2 + s^2 - a_2^2 - d_4^2}{2a_2 d_4}\right)$

    * Another solution is $\theta_3 = \pi - \sin^{-1}\left(\frac{r^2 + s^2 - a_2^2 - d_4^2}{2a_2d_4}\right)$
        - There are 2 configurations possible, the "elbow-up" and "elbow-down" configurations
    * Note we can also write $\theta_3 = \text{atan2}(D, \pm\sqrt{1 - D^2})$ where $D = \frac{r^2 + s^2 - a_2^2 - d_4^2}{2a_2d_4}$
        - This gives a solution in the range $[-\pi, +\pi]$ while the former is in $[0, 2\pi]$
  - Using θ_3, now $\theta_2 = \text{atan2}(s, r) - \text{atan2}\left(d_4 \sin\left(\theta_3 - \frac{\pi}{2}\right), a_2 + d_4 \cos\left(\theta_3 - \frac{\pi}{2}\right)\right)$
  - For inverse orientation kinematics, if we carry out the computation for H_6^3 we get something very similar to the *zyz* Euler rotation matrix, but some of the signs may be different
    * The differences in sign are due to the assignment of the x axes, since they can be flipped and still follow the DH rules
    * Watch out for this in labs