Tutorial 8, Mar 22, 2024

Markov Chains

- Markov property: $P[X_{t+1} = y | X_t = x, X_{t-1} = X_t - 1, \ldots, X_0 = x_0] = P[X_{t+1} = y | X_t = x]$
- Define $P(x, y) = P[X_{t+1} = y | X_t = x]$, i.e. the probability of starting in x and ending in y after one step
- Define the *stochastic matrix* or *transition matrix* $\boldsymbol{P}$ such that $p_{ij} = P(i, j)$
  - Each row sums to 1
- If $X_t \sim \mu_t$, then the next distribution is $P[X_{t+1} = y] = \sum_{x \in \mathcal{X}} \mu_t(x) \boldsymbol{P}(x, y)$ and so $\boldsymbol{\mu}_{t+1} = \boldsymbol{\mu}_t \boldsymbol{P} = \boldsymbol{\mu}_0 \boldsymbol{P}^t$
- The probability of transitioning from x to y in t steps is $P^t(x, y)$
- $\boldsymbol{\pi}$ is *stationary* if $\boldsymbol{\pi} = \boldsymbol{\pi} \boldsymbol{P}$, so if we start in $\boldsymbol{\pi}$ as the initial distribution, the distribution stays constant
- If $\boldsymbol{P}$ is aperiodic and irreducible, then the Markov chain will have a stationary distribution given by $\boldsymbol{\pi}$ above