Lecture 8, Feb 1, 2024

Block Diagrams

- We use block diagrams to depict cause-and-effect relationships within a system
  - Each block shows a function acting on an input to generate output
    - The block is depicted with a transfer function
  - Arrows are used to represent the direction of signals (i.e. information flow)
  - Circles are used for algebraic sum and differences of signals
  - Nodes (aka pick-off points) are used for branching out signals
- Note when we have a feedback system, we usually depict the plant' transfer function by $G(s)$ and the feedback's transfer function by $H(s)$

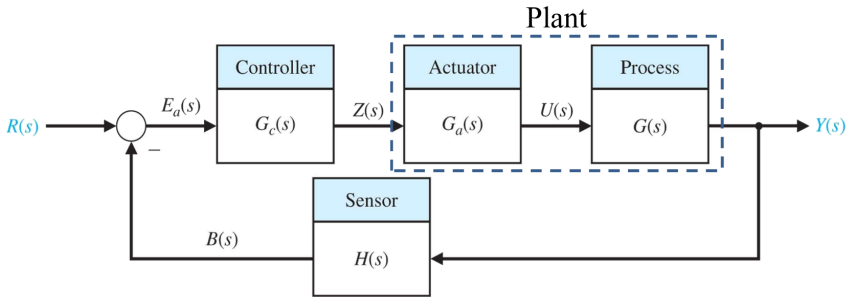

Figure 1: General feedback system.

- Transfer function definitions for a general feedback system:
  - Closed-loop TF: $\mathcal{T} = \dfrac{Y(s)}{R(s)} = \dfrac{G_c(s)G_a(s)G(s)}{1 + G_c(s)G_a(s)G(s)H(s)}$
  - Open-loop TF: $L(s) = \dfrac{B(s)}{E_a(s)} = G_c(s)G_a(s)G(s)H(s)$
    - Note this is not the output to input without feedback (which would be feedforward TF)
    - This is the ratio of the feedback signal to $E_a(s)$
  - Error TF: $\dfrac{E(s)}{R(s)} = \dfrac{R(s) - Y(s)}{R(s)} = \dfrac{1 + G_c(s)G_a(s)G(s)(H(s) - 1)}{1 + G_c(s)G_a(s)G(s)H(s)}$
    - Note the $E(s)$ here is not the same as $E_a(s)$
  - Feedforward TF: $\dfrac{Y(s)}{E_a(s)} = G_c(s)G_a(s)G(s)$
    - Note here we use $E_a(s)$ not $E(s)$
  - Feedback TF: $\dfrac{B(s)}{R(s)} = \dfrac{G_c(s)G_a(s)G(s)H(s)}{1 + G_c(s)G_a(s)G(s)H(s)}$
    - This is the ratio of feedback signal to input signal
    - We can find this easily by taking $\dfrac{Y(s)}{R(s)}H(s)$
  - Sensitivity TF: $\mathcal{S}(s) = \dfrac{1}{1 + G_c(s)G_a(s)G(s)H(s)}$
    - This is important to the robustness of the controller as we will later see
    - This is the inverse of the characteristic equation
  - Characteristic equation: $1 + G_c(s)G_a(s)G(s)H(s)$
    - This is the denominator of the closed-loop TF
- Block diagrams can be simplified to find the overall transfer function of the system
  - There are a number of simplification rules

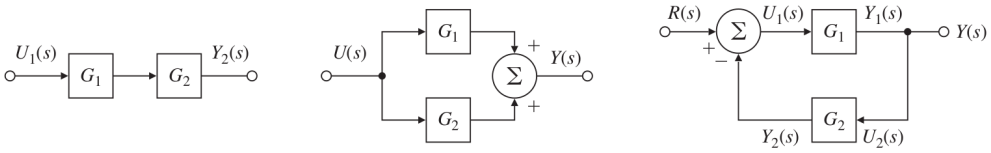

Figure 2: Block diagram reduction rules.

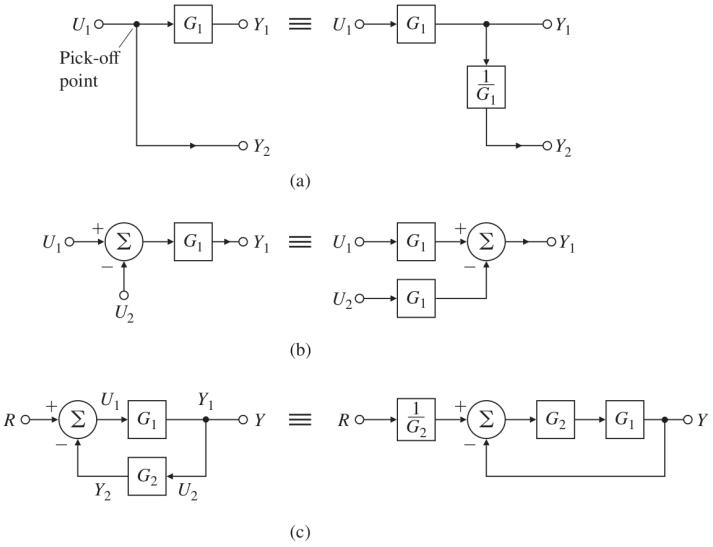

Figure 3: Block diagram reduction rules.