Lecture 26, Apr 12, 2024

Design for Dynamic Compensation

- For PI control, $D_c(s) = 1 + \frac{1}{T_I s}$, the steady-state error of the system is reduced with minimal impact on the bandwidth
  - The gain is high at low frequencies, which reduces the steady-state error
    * This increases the system type
  - However, it causes the reduction of phase margin at frequencies lower than the breakpoint $\frac{1}{T_I}$, which degrades stability
    * This makes sense since we know that an integral controller may destabilize the system
  - We usually place the break point $\frac{1}{T_I}$ at a frequency substantially less (one octave to multiple decades) than the crossover frequency, so that the impact on PM is minimal
  - The main practical problem of PI control is *integral windup* (aka *overflow*), leading to saturation of the system
    * A sudden change in the reference causes the integral term to accumulate too much
    * This leads to a very sluggish controller

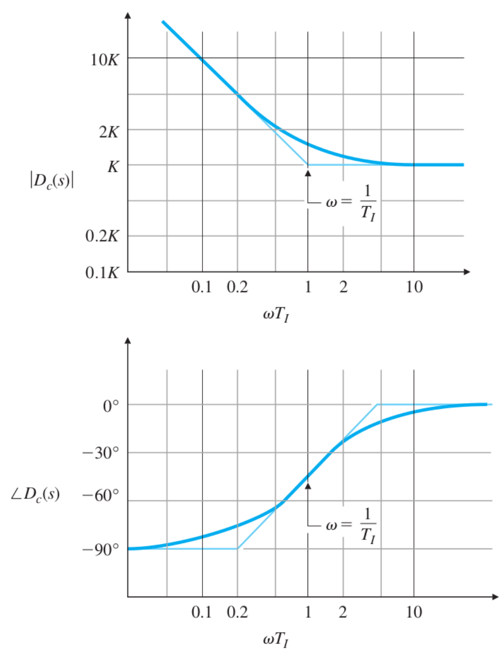

Figure 1: Bode plots of the integral controller.

- We typically use a lag compensator instead, $D_c(s) = \alpha \frac{T_I s + 1}{\alpha T_I s + 1}$, where $\alpha > 1$, so the pole has a lower break point than the zero
  - This can decrease the steady-state error without lowering the crossover frequency
  - The magnitude no longer increases to infinity at lower frequencies and instead converges to α, however the phase at low frequencies now converges to 0, instead of the $-90°$ before
    * This allows us to still reduce the steady-state error, without sacrificing too much phase margin
  - We choose the poles and zero relatively close together, and well below (one octave to multiple decades) the crossover frequency (i.e. choose a large T_I)
    * Having the corner frequencies far from the break point minimizes the reduction in phase margin

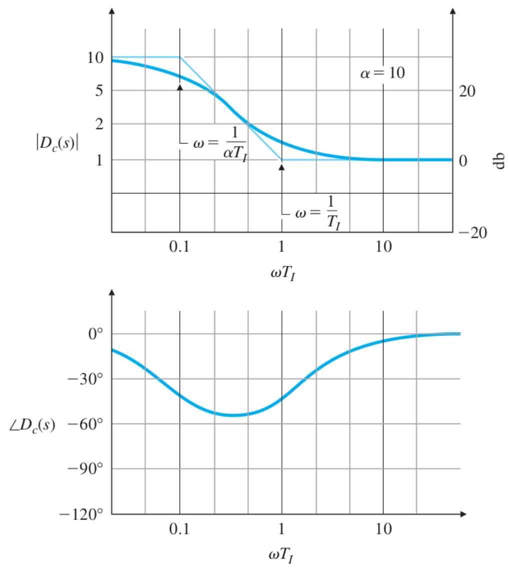

Figure 2: Bode plots of the lag compensator.

- Lag compensator design procedure:
  1. Determine the gain K required to get the desired PM without compensation, with a 5° to 10° margin to account for the PM reduction of the compensator
  2. Draw the Bode plot of the uncompensated open-loop TF and check the low-frequency gain, which gives the steady-state error
  3. Determine the value of α to meet the steady-state error requirement – α is how much more we need to multiply the low-frequency gain by in order to meet the steady-state error requirement
  4. Choose the upper corner frequency $\dfrac{1}{T_I}$ (the zero) to be one octave to multiple decades below the uncompensated ω_c
  5. Iterate on the design and verify that it meets requirements
- Example: $G(s) = \dfrac{115}{(s+1)(s+3)(s+28)}$, design a lag compensator to get an overshoot of less than 15% and a steady-state error of less than 2%
  - $M_P < 15\% \implies PM = 56°$ from the plots; having a margin gives $PM = 66°$
  - We have no restrictions on ω_c so pick it so that we get $PM = 66°$
    * Plot the Bode plot and find that $\omega_c = 2.5$ gives us the desired PM; at this value, the gain is currently 0.38
    * We get $K = 2.63$ to get us the desired ω_c
  - Using this value of K, the uncompensated K_p is 3.6 (same as the value of the magnitude plot at $\omega = 0$)
  - We want $e_{ss} = \dfrac{1}{1+K_p} < 0.02 \implies K_p > 49$ instead; choose $K_p = 50$ for some margin
    * Therefore $\alpha = \dfrac{50}{3.6} = 14$
  - Choose $\dfrac{1}{T_I}$ one decade below the crossover frequency, and double check that requirements are satisfied
- A PID compensator $D_c(s) = K(T_D s + 1)\left(1 + \dfrac{1}{T_I s}\right)$ can be used to improve both transient and steady-state responses
  - Roughly equivalent to combining lead and lag compensators
    * $D_c(s) = \gamma \left(\dfrac{T_D s + 1}{\frac{T_D}{\gamma} s + 1}\right)\left(\dfrac{T_I s + 1}{\gamma T_I s + 1}\right)$ for $\gamma > 1$

- For some systems, the Bode plot will cross over the real axis multiple times; this results from the natural modes of vibration of the system
  - *Gain stabilization* is the simple approach of modifying K to bring the entire plot down
  - *Phase stabilization* is the use of notch compensators that remove the system's response at the problematic frequencies
- A lead-lag compensator, $D_c(s) = \beta\left(\frac{T_I s + 1}{\beta T_I s + 1}\right)\left(\frac{T_D s + 1}{\alpha T_D s + 1}\right)$ for $\alpha < 1, \beta > 1$, combines both
- Example: given $G(s) = \frac{1}{s^2(s+2)}$, design a lead-lag compensator to get $t_r \leq 1$, $M_p \leq 40\%$, $t_s \leq 10$ (for 2%), and $e_{ss} \leq 10\%$
  - Convert: $\omega_n \geq 1.8$, $\zeta \geq 0.3 \implies PM \geq 30°$, $\sigma = \zeta\omega_n \geq 0.46$, $e_{ss} = \frac{1}{K_a} \leq 0.1$
    * The requirement for e_{ss} suggests that a lead-lag compensator is likely needed
  - Initially, choose $\omega_n = 2$ and so $\omega_{BW} \approx 2$; start with crossover frequency at half bandwidth, $\omega_c = 1$, and phase margin of 40° (with margin added)
  - At ω_c, the magnitude $|G(j\omega_c)| = 0.447$ for the uncompensated system, so choose $K = \frac{1}{0.447}$ to make this the crossover frequency
  - The phase is $-207°$ at ω_c, so the initial phase margin is $-27°$ – we need to add $\phi_{max} = 67°$ of phase margin
  - Using the formula, $\alpha = 0.042$
  - Choose $\omega_{max} = \omega_c = 1.0$, so that $D_{c1}(s) = \frac{4.88s + 1}{0.21s + 1}$
    * Now at ω_c the magnitude is 4.86, so reduce K further by this factor to get $K = 0.46$
    * This gives a PM of 40°
  - Plotting the step response shows that the overshoot and settling time meet requirements, but not rise time (by a very small amount)
    * Increase K by a small amount to 0.5, which increases overshoot and allows meeting the rise time requirement
  - The existing steady-state error is 0.25; we need $e_{ss} = \frac{1}{K_a} \leq 0.1$ so $K_a \geq 10$
    * The open-loop gain at small frequencies needs to be increased by a factor of 40, so $\beta = 40$
  - Choose the upper corner frequency at a tenth of ω_c, so $\frac{1}{T_I} = 0.1$, giving $T_I = 10 \implies D_{c2}(s) = 40\frac{10s + 1}{400s + 1}$
    * As expected, the open-loop response is faster with worse overshoot
  - In time domain this now has an overshoot that is slightly over the limit, so we need to iterate:
    * Try $\frac{1}{T_I} = 0.05 \implies T_I = 20 \implies D_{c2}(s) = 40\frac{20s + 1}{800s + 1}$
      - This now doesn't meet the requirements