Lecture 20, Mar 21, 2024

Example: 1-DoF Satellite Attitude Control (Continued)

- Consider the case of collocated control (Θ_1), with the previous lead compensator at $z = 1, p = 12$
  - The characteristic equation is $1 + K\frac{s+1}{s+12}\frac{(s+0.1)^2+6^2}{s^2((s+0.1)^2+6.6^2)} = 0$
  - The flexible mode adds two additional branches, but since it also has two zeros, the two new branches go to the new zeros
  - Even though the 2 new poles are closer to the imaginary axis and have less damping, because they are very close to zeros, they are mostly cancelled out
  - Therefore the response of the system is still mostly dominated by the same two poles as in the double-integrator case
    * The actual response will exhibit very small oscillations (added to the normal response) caused by the flexible modes
      - Since these are almost undamped, they will stay for a very long time
    * If the gain is very large, the dominating poles are now on the asymptote
  - Overall, the single flexible mode brings lightly damped roots
- Note that in the above we assumed that the open-loop zeroes are the same as the closed-loop zeros, which is only true when we have a unity feedback system

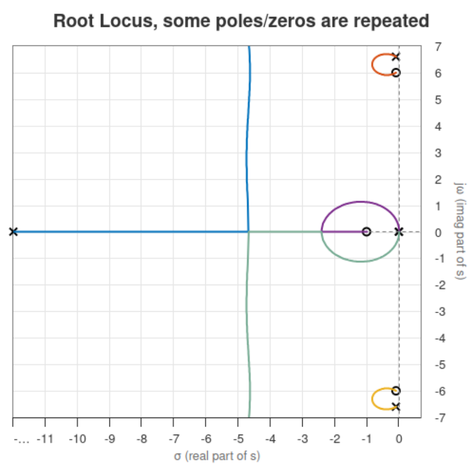

Figure 1: Root locus plot of the collocated case.

- In the non-collocated case (Θ_2), we are missing the two zeros
  - Because we don't have the zeros, the new branches now go to infinity instead of their zeros; the asymptotes make the poles go into the RHP, introducing instability
  - These poles are still barely in the LHP, so the system can still be stable for some gain values, but it is now unstable for larger gains
  - Furthermore these poles are no longer cancelled out by zeros, so they will dominate the system and introduce very high overshoot
  - This is why the non-collocated system is much harder to control

Design for Dynamic Compensation

- Lead compensator: $D_c(s) = K\frac{s+z}{s+p}$ where $z < p$
  - For a sinusoidal input, its output leads the input (output phase shift is positive)
  - Note that due to causality, the output doesn't start earlier than the input; but with a sustained sinusoidal input, the phase shift gradually approaches positive

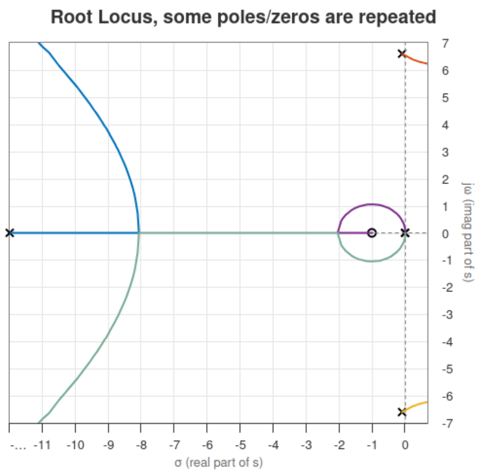

Figure 2: Root locus plot of the non-collocated case.

  - This comes at a cost of some amplitude
  - Approximates PD control; speeds up response (lowering rise time) and decreases overshoot
- Lag compensator: $D_c(s) = K\dfrac{s+z}{s+p}$ where $z > p$
  - For a sinusoidal input, the output lags the input (negative phase shift)
  - The amplitude of the output is now larger than the input
  - Approximates PI control, decreasing steady-state error
- Notch compensator: $D_c(s) = K\dfrac{s^2 + 2\zeta\omega_0 s + \omega_0^2}{(s+\omega_0)^2}$
  - Attenuates the input around some unwanted frequency, acting as a band-stop filter
  - Enhances stability for plants with lightly damped flexible modes (cancels them out)
  - Typically has two complex zeros, which can capture problematic poles
    * Also has two real poles, but typically ω_0 is large, so they are far out in the LHP and usually has little effect
- Note that all 3 compensator do not have any poles at the origin, so the type of the plant is unchanged by adding a compensator
- Consider the example plant $G(s) = \dfrac{1}{s(s+1)}$, e.g. a servo mechanism

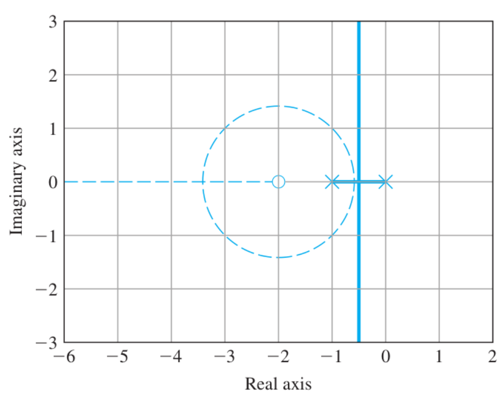

Figure 3: Root locus plots for a P (solid line) and PD (dashed line) controller.

- Example: lead compensation
  - We typically start with the simplest possible controller first

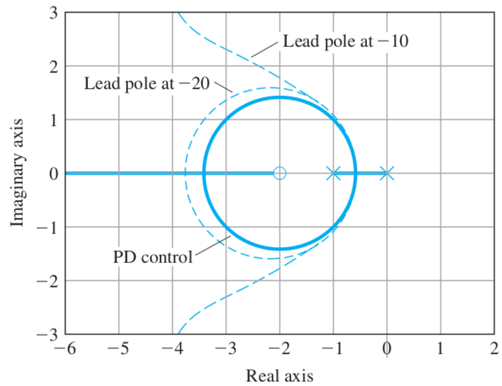

Figure 4: Root locus plots for different lead compensator gains.

- Consider P control: $D_c(s) = K$
  * Since the asymptote is close to the imaginary axis, the damping is very low
  * If we want a certain ω_n for a certain rise time, we will have a large overshoot
  * e.g. for $\omega_n = 2 \implies \zeta = 0.25$
- Now consider PD control: $D_c(s) = K(s + 2)$
  * Now for the same value of ω_n, our poles will be on the circle, and ζ is significantly larger, improving damping without sacrificing speed
  * e.g. for $\omega_n = 2 \implies \zeta = 0.75$
- Now the lead compensator $D_c(s) = K\dfrac{s+2}{s+p}$
  * As we've seen previously, depending on the location of the pole relative to the zero, we can get very different behaviour
  * For small K, the lead compensator approximates PD control well, regardless of where the pole is
  * For large p, the lead compensator also behaves like PD control
  * The additional pole slightly lowers damping (for the same ω_n we see that ζ is smaller)
    - This effect is negligible for low K and large p
  * Typically, we place the zero near the desired closed-loop ω_n (0.25 to 1 times ω_n) and the pole 5 to 25 times the value of the zero
    - The further p is, the closer we get to PD; we get slightly better performance, but noise will increase