Lecture 6, Jan 20, 2023

Fermions and Bosons

- All electrons are identical, so you can't distinguish between them
  - Therefore a mathematically suitable wavefunction must reflect this
  - This is the reason for the Pauli exclusion principle
- Since probabilities are described by $|\Psi|^2$, physically we cannot distinguish between Ψ and $-\Psi$; i.e. if two wavefunctions represent the same quantum state, they must be related through a constant or phase only
  - Since we can't distinguish between two particles, this means that their combined wavefunction must be symmetric or antisymmetric under particle exchange
  - i.e. $\Psi(A, B) = \Psi(B, A)$ or $\Psi(B, A) = -\Psi(A, B)$
  - Particles for which $\Psi(A, B) = \Psi(B, A)$ are called *bosons* (e.g. photons); particles for which $\Psi(B, A) = -\Psi(A, B)$ are called *fermions* (e.g. electrons)
- Electrons are fermions which is why they obey the Pauli exclusion principle: no two electrons can occupy the same quantum state
  - Suppose that electrons A and B do occupy the same quantum state, then $\Psi(A, B) = \Psi(B, A)$
  - However since electrons are fermions we also have $\Psi(B, A) = -\Psi(A, B)$
  - Combining the two we get that $\Psi(A, B) = \Psi(B, A) = 0$, i.e. the probability of two electrons in the same quantum state is zero
- Suitable functions for the PEP must satisfy:
  - Be indistinguishable to electron exchange
  - Normalizable wavefunction
  - Have to include spin
  - Wavefunction must be antisymmetric to electron exchange