# Lecture 23 (2-6), Mar 10, 2023

## Fundamental Postulate of Statistical Mechanics

- In an isolated gas of many particles ($N \sim 10^{23}$) there are many ways we can distribute a fixed amount of energy $E$ amount the particles
  - Each of these ways the energy is distributed is a *microstate*; there are many such microstates
  - The total energy is a *macrostate*
- Collisions lead to a change in the way energy is distributed, leading to randomness
- The fundamental postulate of statistical mechanic says that all accessible microstate is equally likely
  - Accessible microstates are ones that have the right amount of total energy that matches a given macrostate
  - We give up the mechanistic description – it's impractical to know the trajectory of each particle
  - Instead we replace it with the probabilistic distribution
    * As $N$ gets large, probabilities become certainties as probability distribution peaks become sharper
- The *multiplicity function* is the number of microstates accessible to a given macrostate
  - Once you have the multiplicity function of a system, you can deduce everything about the system

## Electronic Paramagnet

- A material that has a macroscopic magnetic moment
- We'll model the microscopic magnetic moments as spins
  - The 2 degrees of freedom are the spins
  - $s_i = \{ +1, -1 \}$ for $i = 1, \cdots, N$
- We ignore spin-spin interaction, spin-atom interaction
  - Without these simplifications this would be extremely hard to solve
  - However without spin-spin interaction this system does not reach equilibrium
- In this $N$-spin system we have $2^N$ possible microstates
  - Microstates are discrete and finite – this is not always the case
- Motivation: If we put a magnet in a magnetic field, it has energy $U = -\mu \cdot \vec{B}$, which is minimized when the 2 vectors are aligned
  - Therefore $U_i = -\mu_0 B s_i$ so $U = -\mu_0 B \sum_{i=1}^{N} s_i = -\mu_0 BS$ where $S$ is the total spin, the sum of the individual spins
- $S$ is the macrostate, which can be observed macroscopically
  - $S$ is an integer in the range $[-N, N]$, in increments of 2 (since flipping a spin changes the total spin by 2)
  - This gives us a total of $N + 1$ possible macrostates
    * Instead of talking about $S$, we just need to specify $N_\uparrow$ since $N_\downarrow = N - N_\uparrow$
    * $S = 2N_\uparrow - N$
    * There are $N + 1$ possible values of $N_\uparrow$ so there are $N + 1$ macrostates
  - $U$, $S$, $N_\uparrow$ can all be used equivalently to specify a macrostate
- For this system, the multiplicity function is easy to calculate
  - $\Omega(N_\uparrow, N) = \binom{N}{N_\uparrow} = \dfrac{N!}{(N - N_\uparrow)! N_\uparrow!}$