Lecture 22 (2-5), Mar 9, 2023

Adiabatic Processes

- In an adiabatic process there is no heat exchange between a system and its surroundings ($Q = 0$)
- Since there is no heat $\Delta U = -p\Delta V$ at every step
- From $U = \dfrac{f}{2}NkT$ (where $f = 3$ for monoatomic gases and $f = 7$ for diatomic gases) we have $\Delta U = \dfrac{f}{2}Nk\Delta T$ so $-p\Delta V = \dfrac{f}{2}Nk\Delta T$
- Bring in the ideal gas law $Nk\Delta T = \Delta(pV) = V\Delta p + p\Delta V$, combine with above $-p\Delta V = \dfrac{f}{2}(V\Delta p + p\Delta V)$
- $-\left(1+\dfrac{f}{2}\right)p\Delta V = \dfrac{f}{2}V\Delta p \implies -\dfrac{1+\frac{f}{2}}{\frac{f}{2}}\dfrac{\Delta V}{V} = \dfrac{\Delta p}{p} \implies \dfrac{2+f}{f}\dfrac{\mathrm{d}V}{V} = \dfrac{\mathrm{d}p}{p}$
- $-\dfrac{2+f}{f}\mathrm{d}\ln V = \mathrm{d}\ln p$, integrate to get $-\dfrac{2+f}{f}\ln\dfrac{V_2}{V_1} = \ln\dfrac{p_2}{p_1}$
- Therefore $\left(\dfrac{V_1}{V_2}\right)^{\frac{2+f}{f}} = \dfrac{p_2}{p_1}$, or $V_1^{\frac{2+f}{f}}p_1 = V_2^{\frac{2+f}{f}}p_2$
  - This means $V_2^{\frac{2+f}{f}}p_2$ is a constant
  - $\dfrac{2+f}{f} = 1 + \dfrac{2}{f} = \gamma > 1$, which means that for the same pressure, the volume will be smaller; or for the same volume, the pressure will be smaller
  - The adiabatic curve on the P-V diagram falls steeper than the isotherm

Heat Capacities

- In general $C = \dfrac{Q}{\Delta T}$ is ill-defined because Q depends on the environment, i.e. how the gas is being heated
- If V is constant then $\Delta U = Q$ since there is no work; therefore we can define a constant volume heat capacity $C_v = \left(\dfrac{\Delta U}{\Delta T}\right)_V = \left(\dfrac{\partial U}{\partial T}\right)_V$
  - For an ideal gas $U = \dfrac{f}{2}NkT$ so $C_V = \dfrac{f}{2}kN$
- If P is constant, Q and U have no straightforward relationship
  - $Q = \Delta U + p\Delta V$
  - $C_p = \left(\dfrac{\Delta U + p\Delta V}{\Delta V}\right)_p = \left(\dfrac{\Delta U}{\Delta T}\right)_p + p\left(\dfrac{\Delta V}{\Delta T}\right)_p$
  - $C_p = \left(\dfrac{\partial U}{\partial T}\right)_p + \left(\dfrac{\partial V}{\partial T}\right)_p$ for any gas in general
  - For the ideal gas $\left(\dfrac{\partial U}{\partial T}\right)_p = \left(\dfrac{\partial U}{\partial T}\right)_V = \dfrac{f}{2}NkT$ are the same since energy only depends on the temperature
  - $\left(\dfrac{\partial V}{\partial T}\right)_p = \left(\dfrac{\partial}{\partial T}\dfrac{NkT}{p}\right)_p = \dfrac{Nk}{p}$
  - Combing these we get $C_p = \dfrac{f}{2}Nk + Nk = \dfrac{f+2}{2}Nk = \dfrac{f+2}{2}C_v = \gamma C_v$
  - $C_p > C_v$ always holds true for any gas
    - For an ideal gas it is because some of the energy goes towards the expansion of the gas
    - This also works for everything else even if it contracts when being heated

Directional Asymmetry of Time

- e.g. heat only flows from hot to cold, diffusion only happens from dense to diffuse

- This is due to the randomness inherent to a system of many bodies
- There is one unifying principle, the fundamental postulate of statistical mechanics: Consider a closed (constant energy, isolated) system of many particles, in the system every accessible microstate is equally likely