Lecture 9, Jan 30, 2023

Independence of Random Variables

Definition

Let X and Y be two random variables with joint probability distribution $f(x, y)$ and marginal distributions $g(x), h(y)$; these random variables are *independent* if $f(x, y) = g(x)h(y)$

- This definition applies to both continuous and discrete cases

Expectation

Definition

Let X be a random variable with distribution $f(x)$; the expectation value of X is, in the discrete case:

$$E[x] = \sum_x x f(x)$$

in the continuous case:

$$E[x] = \int_{-\infty}^{\infty} x f(x) \, \mathrm{d}x$$

- Also known as expected value or the mean, denoted by μ
- We may replace x with a function of the random variable $g(x)$ to find its expectation

Definition

Let X and Y be random variables with joint distribution function $f(x, y)$; the expectation value of the function $g(X, Y)$ is, in the discrete case:

$$E[g(X, Y)] = \sum_x \sum_y g(x, y) f(x, y)$$

in the continuous case:

$$E[g(X, Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f(x, y) \, \mathrm{d}x \, \mathrm{d}y$$

- Expectation values easily generalize to multiple variables