Lecture 7, Jan 23, 2023

Probabilities of Discrete Random Variables

Definition

$f(x)$ is the *probability mass function* (PMF) of a discrete random variable X if:

- $f(x) = P(X = x), \forall x \in S$
- $\displaystyle\sum_{x \in S} f(x) = 1$
- $f(x) \geq 0, \forall x \in S$

Definition

The *cumulative distribution function* (CDF) of a discrete random variable X with PMF $f(x)$ is

$$F(x) = \sum_{t \leq x} f(t)$$

or equivalently

$$F(x) = P(X \leq x)$$

- e.g. flipping a coin 3 times:
  - $f(x) = \begin{cases} \frac{1}{8} & x = 0, x = 3 \\ \frac{3}{8} & x = 1, x = 2 \end{cases}$
  - $F(-1) = 0, F(0) = \frac{1}{8}, F(1) = \frac{1}{2}, F(2) = \frac{7}{8}, F(3) = 1$
  - $P(X \geq 2) = 1 - P(X \leq 1) = 1 - F(1) = \frac{1}{2}$
- Properties of CDFs:
  - $F(-\infty) = 0$
  - $F(\infty) = 1$
  - All CDFs are nondecreasing

Probabilities of Continuous Random Variables

- In the continuous case, the probability of the random variable equalling any specific value is zero, since there are an uncountably infinite number of outcomes in every interval
- For a continuous random variable we can only talk about probabilities of the variable being in some interval

Definition

$f(x)$ is the *probability distribution/density function* (PDF) of a continuous random variable X if:

- $\displaystyle\int_a^b f(x)\,\mathrm{d}x = P(a \leq X \leq B), \forall a \leq b \in \mathbb{R}$
- $\displaystyle\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = 1$
- $f(x) \geq 0, \forall x \in S$

Definition

The *cumulative distribution function* (CDF) of a continuous random varaible X with PDF $f(x)$ is

$$F(x) = \int_{-\infty}^{x} f(t)\,\mathrm{d}t$$

or equivalently

$$F(x) = P(X \leq x)$$

- This gives $P(a \leq X \leq b) = F(b) - F(a)$
- The properties of CDFs from the discrete case carry over