Lecture 6, Jan 20, 2023

Bayes' Rule and Total Probability

- Given a partition of A into $C_1, \cdots, C_k$
- $$\begin{aligned} P(B|A) &= \frac{P(B)P(A|B)}{P(A)} \\ &= \frac{P(B)P(A|B)}{\sum_{i=1}^{k} P(A \cap C_i)} \\ &= \frac{P(B)P(A|B)}{\sum_{i=1}^{k} P(A|C_i)P(C_i)} \end{aligned}$$
  - Often $B = C_n$ for some $n = 1, \cdots, k$

Random Variables

Definition

A *random variable* is a function that maps each element of a sample space to a real number

- We denote a random variable with capital letters, e.g. X, Y
- In the discrete case, the random variable can only take on a finite (or countably infinite) set of values
- In the continuous case the random variable can take any value in the real numbers
- We write $X = x$, with the lowercase x to denote values that the random variable can take on
- Example: coin flips
  - $S = \{H, T\}$; our random variable can be $X = \begin{cases} 0 & H \\ 10 & T \end{cases}$
  - If we do 3 coin flips, X can be the number of heads