Lecture 4, Jan 16, 2023

Probability of Unions

Equation

The probability of the union of two events $P(A \cup B)$ is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- In general $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - $P(A \cup B)$ is the overlap of A and B, which is counted twice by $P(A) + P(B)$
- Example: Probability of rolling a 7 or having at least one 2
  - Probability of rolling a 7 is $\frac{1}{6}$
  - Probability of rolling at least one 2 is $\frac{11}{36}$
  - Probability of rolling a 7 and having at least one 2 is $\frac{2}{36}$ (2 and 5, or 5 and 2)
  - Final probability is $\frac{15}{36} = \frac{5}{12}$
- For the union of more than 2 events, we can expand the rule:
  - $$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B \cup C) + P(A \cap (B \cup C)) \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P((A \cap B) \cup (A \cap C)) \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C) \end{aligned}$$
  - Note intersection distributes over union

Conditional Probability

Definition

If A and B are both events, the probability of B *conditioned* on A (or probability of B given A), $P(B|A)$ is the probability of B given that A occurs

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- This formula arose due to $P(A \cap B) = P(A)P(B|A)$, i.e. the probability of both A and B happening is the probability of A happening times the probability of B happening given that A happens