# Lecture 2, Jan 11, 2023

## Counting

- Counting is the problem of finding the number of elements in some event $A$
  - e.g. for a coin flip, if $A = \{ H \}$, then we have one element; for a die, if $A = \{ \text{even} \}$, then we have 3 elements
- Two events $A$ and $B$ are *mutually exclusive* if $A \cap B = \varnothing$
  - For two events that are mutually exclusive, we can add up their number of elements when counting

## Multiplying Options

- Where we can choose 1 option from each category, we multiply the category sizes together
  - e.g. choosing a president and VP from $n$ people has $n(n-1)$ possibilities
- Example: How many even 4-digit numbers can we make from $\{ 0, 1, 2, 5, 6, 9 \}$?
  - Consider events $A$ and $B$:
    * In $A$, 0 is the last digit so $A$ has $1 \cdot 5 \cdot 4 \cdot 3 = 60$ elements
    * In $B$, 0 is not the last digit ($A$ and $B$ are mutually exclusive); the last digit could be 2 or 6 and the first digit can be anything but zero or what we chose for the last digit, so we have $2 \cdot 4 \cdot 4 \cdot 3 = 96$ elements
    * Since $A \cap B = \varnothing$, the total count is 156

## Permutations

- A *permutation* is an ordering of the elements in an event
- Given $n$ items, there are $n!$ permutations
- If we want to permute $r$ items out of $n$, there are $\dfrac{n!}{(n-r)!}$ permutations
- With $n$ slots to fill, where there are $m$ kinds of items and $n_k$ of each item, the number of permutations is $\dbinom{n}{n_1, n_2, \cdots, n_m} = \dfrac{n!}{n_1! n_2! \cdots n_m!}$
- Example: How many distinct ways can we order "ATLANTIC"?
  - $\dbinom{8}{2, 2, 1, 1, 1, 1} = \dfrac{8!}{2!2!1!1!1!1!} = 10080$
- Example: If we flip a coin 10 times, how many sequences have 4 heads?
  - We're looking at combinations of $HHHHTTTTTT$, so $\dbinom{n = 10}{n_1 = 4, n_2 = 6} = \dfrac{10!}{4!6!} = 210$
  - This gives us a probability of getting 4 heads of $\dfrac{210}{2^1 0} = \dfrac{210}{1024}$ (assuming a fair coin)