Lecture 18, Feb 27, 2023

Application: Renewable Energy & Electricity Markets

- Renewable energy operators are paid for power produced
- Penalties for uncertainty in power production
- Basic quantities:
  - At the start of the hour, wind forecasts power $\hat{p}$
  - Actually produces power p
  - Market price λ
  - Fees u^+ for overproduction, u^- for underproduction
  - Total revenue is then $\tilde{J} = \lambda p - u^-(\hat{p} - p)^+ - u^+(p - \hat{p})^+$ where $(\cdot)^+ = \max(\cdot, 0)$
- Suppose p is distributed according to $f(p)$ and CDF $F(P)$; we want to maximize $J = E_p[\tilde{J}]$ over $\hat{p}$
- $E_p[\tilde{J}] = \lambda \int_{-\infty}^{\infty} p f(p) \,\mathrm{d}p - u^- \int_{-\infty}^{\hat{p}} (\hat{p} - p) f(p) \,\mathrm{d}p - u^+ \int_{\hat{p}}^{\infty} (p - \hat{p}) f(p) \,\mathrm{d}p$
- $\dfrac{\mathrm{d}J}{\mathrm{d}\hat{p}} = -u^- \left(\int_{-\infty}^{\hat{p}} f(p) \,\mathrm{d}p + (\hat{p} - \hat{p}) f(p) \right) - u^+ \left(\int_{\hat{p}}^{\infty} -f(p) \,\mathrm{d}p - (\hat{p} - \hat{p}) f(p) \right)$

  $= -u^- F(\hat{p}) + u^+(1 - F(\hat{p}))$
- Solve for $\hat{p}$: $-u^- F(\hat{p}) + u^+(1 - F(\hat{p})) = 0 \implies F(\hat{p}) = \dfrac{u^+}{u^- + u^+} \implies \hat{p} = F^{-1}\left(\dfrac{u^+}{u^- + u^+}\right)$
  - We know F is invertible, because by definition F is non-decreasing
- What does this tell us?
  - F caps at 1, so if $u^+ \gg u^-$, the ratio is close to 1, therefore $\hat{p} \to F^{-1}(1) = \infty$
    * If the penalty for overproducing is way bigger than the penalty for underproducing, then it's better to forecast a very big number
  - Conversely $u^+ \ll u^- \implies \hat{p} \to F^{-1}(0) = -\infty$
    * If the penalty for underproducing is way bigger than the penalty for overproducing, then it's better to forecast a very small number
  - If $u^- = u^+$ then $\hat{p} = F^{-1}\left(\dfrac{1}{2}\right)$ which is the *median* of the distribution of p
- This is classically known as the newsvendor problem