Lecture 1, Jan 9, 2023

Uncertainty

- Sources:
  - Limited measurements – more measurements = less uncertainty, e.g. processes that are difficult to measure
  - Difficult-to-model scenarios, e.g. weather, dice roll
- Probability allows us to be systematic in the face of uncertainty ("be consistent about what we don't know")

The Coin Flip

- Heads or tails
- A fair coin has $P(H) = 0.5 = P(T)$; unbalanced coins have different probabilities, but we must have $P(H) + P(T) = 1$ because *something* has to happen 100% of the time
- Example: $P(H) = 0.3, P(T) = 0.7$, then:
  - $P(HH) = P(H)P(H) = 0.09$
  - $P(HT) = P(H)P(T) = 0.21 = P(T)P(H) = P(TH)$
    - This is only true because of *independence*, as the two flips are uncorrelated
  - $P(HT \text{ or } TH) = P(HT) + P(TH) = 0.42$
  - $P(HT, TH, HH \text{ or } TT) = 1$
  - We can construct a fair coin by setting "heads" to HT and "tails" to TH and ignoring all other outcomes

Core Concepts

Definition

The *sample space* S is the set of all possible outcomes

- e.g. for a single coin flip, $S = \{ H, T \}$; for two coin flips, $S = \{ TH, TT, HH, HT \}$

Definition

An *event* is a subset of a sample space, i.e. some subset of possible outcomes

The *complement* of an event A is $A' = \{ a \mid a \in S, a \notin A \}$, the set of everything in S that is not in A

- e.g. $\{ 1, 2, 3, 4, 5, 6 \}$ or $\{ \text{even}, \text{odd} \}$ could both be the set of all events for a die
- Example: $S = \{ (x, y) \mid x^2 + y^2 \leq 1 \}$ (the unit circle), $A = \{ (x, y) \mid (x, y) \in S, x \geq 0 \}$, then $A' = \{ (x, y) \mid (x, y) \in S, x < 0 \}$

Definition

The *intersection* of 2 events A and B, $A \cap B$, is everything in S that is in both A and B

The *union* of 2 events $A \cup B$ is everything in A or B

- e.g. for a die: $\{ \text{even} \} \cap \{ n \leq 3 \} = \{ 2 \}$
- $A \cap A' = \varnothing, A \cup A' = S$