Lecture 8, Jan 25, 2023

Electric Scalar Potential

- By bringing repelling charges together or attracting charges apart, we do work that is stored; this is the idea of *electric potential*

Definition

The electric scalar potential, or voltage ΔV between two points is defined as the work done by an external agent per unit charge, or

$$\Delta V = V_2 - V_1 = V_{21} = -\int_{P_1}^{P_2} \vec{E} \cdot \mathrm{d}\vec{l}$$

- In the case where $\vec{E}$ is constant, we just have ΔV being the field strength times distance between the two points
- Note the negative sign: if the electric field does work between the two points, the potential difference is negative; the electric field always points from high potential to low potential
- Consider a point charge Q at the origin and two points P_1 and P_2
  - $\Delta V = -\int_{P_1}^{P_2} \vec{E} \cdot \mathrm{d}\vec{l} = -\int_{P_1}^{P_2} \frac{Q}{4\pi\varepsilon_0 R^2} \hat{a}_R \cdot \mathrm{d}\vec{l}$
  - We can choose our path so that we move radially first, and then move along a sphere; this allows us to get rid of the dot product, because the radial movement is parallel to $\hat{a}_R$ and the spherical movement is perpendicular
  - We get $\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$ as the potential difference between two points due to a single point charge
  - If we let $R_1 \to \infty$ be our reference, then we just get $\Delta V = V_2 = \frac{Q}{4\pi\varepsilon_0 R_2}$

Definition

The *absolute electric potential* due to a point charge is

$$V(R) = \frac{Q}{4\pi\varepsilon_0 R}$$

This assumes a reference of a charge at $R = \infty$ having zero potential

- Note the expression for the potential is the same as Coulomb's law but the R term is not squared
- A surface which has the same value of V over the entire surface is called an *equipotential surface*
  - This could be a physical surface or an imaginary surface
  - e.g. a sphere surrounding a point charge is an equipotential surface since potential depends only on R; for a dipole these are ellipsoids
  - All perfect conductors are equipotential surfaces
  - The electric field is always perpendicular to equipotential surfaces