Lecture 6, Jan 20, 2023

Gauss's Law

Equation

Electrostatics is governed by two fundamental postulates:
In differential form:

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{D} = \rho_v$$

In integral form:

$$\oint_C \vec{E} \cdot \mathrm{d}\vec{l} = 0$$

$$\oiint_S \vec{D} \cdot \mathrm{d}\vec{S} = Q_{enc}$$

- Electrostatics deal with systems with stationary charges; we represent the field with $\vec{E}$ or $\vec{D}$
  - $\vec{E}$ is the electric field density with units of $\mathrm{V/m} = \mathrm{N/C}$
  - $\vec{D}$ is the electric flux density with units of $\mathrm{C/m^2}$
- $\vec{E}$ and $\vec{D}$ are related by the parameter ε: $\vec{D} = \varepsilon_r \varepsilon_0 = \varepsilon \vec{E}$
  - ε is the *electrical permittivity* of the material
  - ε_r is the *relative permittivity* of the material
  - "Free space" is what you would get in a vacuum $\varepsilon_r = 1$, so $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F/m}$; this is similar to in air where $\varepsilon_r = 1.0006$
- *Gauss's Law* is one of the postulates: $\vec{\nabla} \cdot \vec{D} = \rho_v$
  - In differential form this tells us that at any given point, the divergence of the electric flux density is the same as the volume charge density
- Applying the divergence theorem gives us $\oiint_s \vec{D} \cdot \mathrm{d}\vec{S} = \iiint_V \rho_v \mathrm{d}V = Q_{enc}$
  - In integral form this tells us that the net electric flux through a closed surface is the net charge enclosed by the surface
- Coulomb's Law can be derived from this, if we assume $\vec{D}$ only has a radial component