Lecture 4, Jan 16, 2023

Example: Electric Field Above a Charged Disk

- Charged disk of radius a with total charge Q, measured at a point $P(0, 0, h)$
  1. $\vec{E}_{tot} = \iint \mathrm{d}\vec{E} = \iint_S \frac{(\vec{R} - \vec{R}')}{4\pi\varepsilon_0 \|\vec{R} - \vec{R}'\|^2} \,\mathrm{d}Q'$
  2. $\mathrm{d}Q'$ is the differential charge, $\rho_s \,\mathrm{d}s'$; in this case $\mathrm{d}s = \mathrm{d}s_z$ so $\mathrm{d}Q' = \rho_s r' \,\mathrm{d}\phi' \mathrm{d}r' = \frac{Q}{\pi a^2} r' \,\mathrm{d}\phi' \mathrm{d}r'$
  3. $\vec{R} = h\hat{a}_z$; $\vec{R}' = r'\hat{a}_r$ so $\vec{R} - \vec{R}' = -r'\hat{a}_r + h\hat{a}_z = -r' \cos\phi' \hat{a}_x - r' \sin\phi' \hat{a}_y + h\hat{a}_z$
  4. $\iint \mathrm{d}\vec{E} = \int_0^a \int_0^{2\pi} \frac{\frac{Q}{\pi a^2} r'}{4\pi\varepsilon_0 \left((r')^2 + h^2\right)^{\frac{3}{2}}} (-r' \cos\phi' \hat{a}_x - r' \sin\phi' \hat{a}_y + h\hat{a}_z) \mathrm{d}\phi' \mathrm{d}r'$
  5. The disk is symmetric about the z axis, so there will only be a z component in the total field
  6. $\iint_S \mathrm{d}\vec{E} = \frac{Qh\hat{a}_z}{4\pi(\pi a^2)\varepsilon_0} \int_0^a \int_0^{2\pi} \frac{r'}{((r')^2 + h^2)^{\frac{3}{2}}} \,\mathrm{d}\phi' \mathrm{d}r' = \frac{\rho_s}{2\varepsilon_0} \left(\frac{h}{|h|} - \frac{h}{\sqrt{a^2 + h}} \right) \hat{a}_z$
- In general the steps are:
  1. Select a coordinate system
  2. Find $\mathrm{d}Q'$
  3. Find $\vec{R}, \vec{R}'$ and $\vec{R} - \vec{R}'$
  4. Write out the integral
  5. Look for symmetry before evaluating the integral!
  6. Integrate
- Note if we take $r \to \infty$ we effectively have an infinite plate of charge, then we get $\vec{E}_{tot} = \pm \frac{\rho_s}{2\varepsilon_0} \hat{a}_z$