Lecture 34, Apr 10, 2023

Example: Transformers

- From a magnetic circuit perspective we have 2 sources, $N_1 i_1$ and $N_2 i_2$, and a resistance $R_c = \frac{l_c}{\mu_r \mu_0 S}$
- We obtain $N_1 i_1 - N_2 i_2 = R_c \Phi_{tot}$ by KVL
- Making the approximation that $\mu_r \to \infty$ we get the relation for an ideal transformer: $\frac{i_1}{i_2} = \frac{N_2}{N_1}$
- In reality, the core will not be perfect and there will be flux leakage so this relation is not exact
- With higher frequencies this becomes more noticeable, and we also see a phase shift in the output
  - We have losses in the wire resistances, hysteresis loss, eddy current losses, self-inductances (which become problematic at higher frequencies), etc
- The transformer becomes less ideal as frequency increases, with reduced output amplitude and increased phase shift
- Idealized formulas work fine for power distribution systems which are typically 60 Hz, but at higher frequencies approximations fall apart

Eddy Currents

- Changing $\vec{B}$ leads to a changing $\vec{E}$, which will induce a $\vec{J}$ in a conducting material
- These are referred to as "eddy currents" since they circulate
- Consider an applied field $\vec{B}(t) = B_0 \cos(\omega t)\hat{a}_z$ on a cylinder made of a lossy material with conductivity σ
  - $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = B_0 \omega \sin(\omega t)\hat{a}_z$
  - $\frac{1}{r}\left(\frac{\partial}{\partial r}(rE_\phi) - \frac{\partial E_r}{\partial \phi}\right) = B_0 \omega \sin(\omega t)\hat{a}_z$
  - We can deduce $\frac{\partial E_r}{\partial \phi} = 0$ and so $\vec{E} = \frac{B_0 \omega r \sin(\omega t)}{2}\hat{a}_\phi$
  - This gives us eddy currents $\vec{J} = \frac{\sigma B_0 \omega r \sin(\omega t)}{2}\hat{a}_\phi$
- These eddy currents generate fields of their own that oppose the original field; this causes the effect of a magnet falling slower in a metallic tube
  - This can be used in applications such as frictionless braking
  - However one disadvantage is that the braking force reduces as the speed slows, since the braking force is proportional to the rate of change of the field