Lecture 30, Mar 29, 2023

Magnetic Circuits

- In a magnetic circuits, sources are MMF, $V_m = NI_0$ where I_0 is the current and N is the number of loops
  - The direction of the winding determines the direction of the source
- Reluctance $R = \frac{L}{\mu S}$ is analogous to resistance in an electric circuit
- The magnetic flux Φ_i through the cross-section is analogous to electric current
- Permeability μ is analogous to conductivity σ in an electric circuit
- The usual laws of circuits apply:
  - $\sum_j V_{mj} = \sum_k R_k \Phi_k$ for a loop, like KVL
  - $\sum_i \Phi_i = 0$ for a node, like KCL
- Using the normal circuit analysis techniques (nodal, mesh analysis), we can find all fluxes
- To find the field, we assume a constant B in each cross section, so $B = \frac{\Phi}{S}$

Self and Mutual Inductance

- The field created by a current can cause fluxes in its own loop, or other loops

Definition

Inductance is defined as the amount of flux produced by a source per unit source,

$$L = \frac{\Lambda}{I} = \frac{N\Phi}{I}$$

where $\Lambda = N\Phi$ is the *flux linkage*; it is the dual of capacitance

Both self and mutual inductance exist, with

$$L_{ab} = \frac{N_b \Phi_{ab}}{I_a}$$

denoting the inductance in b caused by a

- Mutual inductances $L_{12} = L_{21}$
- Example: self-inductance of a toroid, magnetic core with N cores carrying current I_0
  - First find the current, from there find the field, then flux, then inductance
  - Using Ampere's law with a contour aligned with the field, $2\pi r B = \mu_r \mu_0 N_1 I_1 \implies \vec{B} = \frac{\mu_r \mu_0 N_1 I_1}{2\pi r}\hat{a}_\phi$
  - Integrate across the cross-section, $\Phi_{11} = \iint \vec{B}_1 \cdot \mathrm{d}\vec{s}_1 = \int_0^h \int_a^b \frac{\mu_r \mu_0 N_1 I_1 \hat{a}_\phi}{2\pi r} \cdot \hat{a}_\phi \,\mathrm{d}r\,\mathrm{d}z$
  - Result: $L_{11} = \frac{\mu_r \mu_0 N_1^2 h}{2\pi} \ln \frac{b}{a}$
  - Notice:
    * More turns directly leads to greater inductance
    * Greater area also leads to greater inductance
    * This only depends on geometry and material, never the current, etc