Lecture 3, Jan 13, 2023

Continuous Charge Distributions

- What is the electric field due to a charged plate?
  - Consider a point at $P(0,0,z)$, and a plate with total charge Q and area A on the xy plane
  - Break the plate into pieces, by superposition $\vec{E}_{tot} = \sum_{i=1}^{N} \vec{E}_i = \sum_{i=1}^{N} \frac{Q_i(\vec{R} - \vec{R}'_i)}{4\pi\varepsilon_0 \|\vec{R} - \vec{R}'_i\|^2}$
  - As $N \to \infty$, the summation becomes an integral and Q_i become $\mathrm{d}Q'$, which are point charges
    * Note primes denote source charge
  - Define the *charge density* ρ_s, in this case with area $\mathrm{C/m^2}$ and $\rho_s = \frac{Q}{A}$
  - $\vec{E}_{tot} = \iint_S \frac{(\vec{R} - \vec{R}')}{4\pi\varepsilon_0 \|\vec{R} - \vec{R}'\|^3} \,\mathrm{d}Q'$
    * $\vec{R}'$ is a function of x and y
- Instead of considering discrete (point charges), which are confined to an infinitesimally small point, in most practical problems charge is distributed in one or more dimensions

Definition

There are 3 types of *continuous charge distributions*:

- Linear: $Q = \int \rho_l \,\mathrm{d}l$

  $$\rho_l = \frac{Q}{L}$$
- Surface: $Q = \iint_S \rho_s \,\mathrm{d}S$

  $$\rho_s = \frac{Q}{A}$$
- Volume: $Q = \iiint_V \rho_v \,\mathrm{d}V$

  $$\rho_v = \frac{Q}{V}$$

In each case ρ denotes the charge density, and the subscript denotes the dimensionality

Differential Elements in Orthogonal Coordinate Systems

- In Cartesian coordinates:
  - Differential lengths are $\mathrm{d}x, \mathrm{d}y, \mathrm{d}z$, so a differential length vector is $\mathrm{d}\vec{l} = \mathrm{d}x\,\hat{a}_x + \mathrm{d}y\,\hat{a}_y + \mathrm{d}x\,\hat{a}_z$
  - Differential surface vectors are $\begin{cases} \mathrm{d}\vec{s}_x = \mathrm{d}y\,\mathrm{d}z\,\hat{a}_x \\ \mathrm{d}\vec{s}_y = \mathrm{d}x\,\mathrm{d}z\,\hat{a}_y \\ \mathrm{d}\vec{s}_z = \mathrm{d}x\,\mathrm{d}y\,\hat{a}_z \end{cases}$
  - Differential volume is $\mathrm{d}V = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$
- In cylindrical coordinates:
  - Differential lengths are $\mathrm{d}r, r\,\mathrm{d}\phi, \mathrm{d}z$
  - Differential length vector is $\mathrm{d}\vec{l} = \mathrm{d}r\,\hat{a}_r + r\,\mathrm{d}\phi\,\hat{a}_\phi + \mathrm{d}z\,\hat{a}_z$
  - Differential surface vectors are $\begin{cases} \mathrm{d}\vec{s}_r = r\,\mathrm{d}\phi\,\mathrm{d}z\,\hat{a}_r \\ \mathrm{d}\vec{s}_\phi = \mathrm{d}r\,\mathrm{d}z\,\hat{a}_\phi \\ \mathrm{d}\vec{s}_z = r\,\mathrm{d}r\,\mathrm{d}\phi\,\hat{a}_z \end{cases}$
    * $\mathrm{d}\vec{s}_r$ represents the cylindrical wall
    * $\mathrm{d}\vec{s}_\phi$ represents a vertical plane coming out of the z axis
    * $\mathrm{d}\vec{s}_z$ represents a horizontal plane

- Differential volume is $\mathrm{d}V = r\,\mathrm{d}r\,\mathrm{d}\phi\,\mathrm{d}z$

- In spherical coordinates:
  - Differential lengths are $\mathrm{d}R, R\,\mathrm{d}\theta, R\sin\theta\,\mathrm{d}\phi$
  - Differential length vector is $\mathrm{d}\vec{l} = \mathrm{d}R\,\hat{a}_R + R\sin\theta\,\mathrm{d}\phi\,\hat{a}_\phi + R\,\mathrm{d}\theta\,\hat{a}_\theta$
  - Differential surface vectors are $\begin{cases} \mathrm{d}\vec{s}_R = R^2\sin\theta\,\mathrm{d}\phi\,\mathrm{d}\theta\,\hat{a}_R \\ \mathrm{d}\vec{s}_\phi = R\,\mathrm{d}\theta\,\mathrm{d}R\,\hat{a}_\phi \\ \mathrm{d}\vec{s}_\theta = R\sin\theta\,\mathrm{d}\phi\,\mathrm{d}R\,\hat{a}_\theta \end{cases}$
  - Differential volume is $\mathrm{d}V = R^2\sin\theta\,\mathrm{d}R\,\mathrm{d}\phi\,\mathrm{d}\theta$