Lecture 29, Mar 27, 2023

Magnetic Circuits: Example

- Example: toroid with N windings of wire carrying current I_0, with a piece cut out from it; what is $\vec{B}$ in that gap?
  - Boundary conditions have to be satisfied at the gap
  - Ignoring fringing effects, we would only have a B_n, so since B_n stays the same across a boundary, $\vec{B}$ also stays the same
  - If the current creates a field with intensity B_0, then $H_{\text{core}} = \dfrac{B_0}{\mu_{rc}\mu_0}, H_{\text{air}} = \dfrac{B}{\mu_0}$
    - Typically $H_{\text{core}} \ll H_{\text{air}}$
  - Apply Ampere's law with a contour aligned with the field: $\oint_C \vec{H} \cdot \mathrm{d}\vec{l} = H_{\text{core}}L_{\text{core}} + H_{\text{air}}L_{\text{air}} = NI_0$
    - Since we have a toroid, we take L to be the mean distances
  - This gives $B_0 = \dfrac{NI_0}{\frac{L_c}{\mu_r\mu_0} + \frac{L_g}{\mu_0}}$
  - To get flux, approximate as $\Phi = \iint_S \vec{B} \cdot \mathrm{d}\vec{s} = B_0 S = \dfrac{NI_0}{\frac{L_c}{\mu_r\mu_0 S} + \frac{L_g}{\mu_0 S}}$
- We can interpret NI_0 as a "voltage" of sorts, and the terms $\dfrac{L_c}{\mu_r\mu_0 S}$ and $\dfrac{L_g}{\mu_0 S}$ to be like "resistances"; this way we can think of this as a magnetic circuit, with "current" being the flux
- NI_0 is V_{mmf}, or the *magnetomotive force* (MMF); $\dfrac{L_c}{\mu_r\mu_0 S}$ is R, or the *reluctance*
  - MMF is the driving force in the same way voltage (electromotive force, EMF) is the driving force in an electric circuit
  - Reluctances resist the flux
  - In this case the reluctance of the core is much smaller than the reluctance of the air gap