Lecture 28, Mar 24, 2023

Magnetic Field Boundary Conditions

- Like the case with the electric field, we wish to find the boundary conditions for the tangential and normal $\vec{B}$ fields across a boundary, between medium μ_1 and medium μ_2, with the normal pointing from medium 2 to medium 1
- We can apply Gauss's law $\oiint_S \vec{B} \cdot \mathrm{d}\vec{s} = 0$ to an infinitely short cylinder right on the boundary, we can conclude that $B_{n1} - B_{n2} = 0 \implies B_{n1} = B_{n2}$
  - In terms of magnetic field intensity, $\vec{B} = \mu_r \mu_0 \vec{H} \implies \mu_{r1} H_{n1} = \mu_{r2} H_{n2}$
- For the tangential fields we can use an Amperian loop with width ΔL right on the boundary, so $\oint_C \vec{H} \cdot \mathrm{d}\vec{l} = H_{t2}\Delta L - H_{t1}\Delta L = I_{enc} = J_s \Delta L$ where J_s is the surface current density on the boundary
  - This gives us $H_{t2} - H_{t1} = J_s$ or more formally $\hat{n}_2 \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$

Summary

Boundary conditions for magnetic fields across two mediums (with surface normal pointing from material 2 to material 1): For the normal component:

$$B_{n1} = B_{n2} \implies \mu_{r1} H_{n1} = \mu_{r2} H_{n2}$$

For the tangential component:

$$H_{t2} - H_{t1} = J_s$$

for a J_s normal to the tangential component, or

$$\hat{n}_2 \times (\hat{H}_1 - \hat{H}_2) = \vec{J}_s$$

- Due to this, as the field travels from a material with a low μ to one with high μ, it will be bent towards the surface (tangential component becomes larger)
  - For ferromagnetic materials μ_r can be really high, so the field becomes essentially entirely tangential
  - This can be used to perform magnetic shielding