Lecture 27, Mar 22, 2023

Generalized Ampere's Law

- Ampere's law becomes $\oint_C \vec{B} \cdot \mathrm{d}\vec{l} = \mu_r \mu_0 I_{enc}$
  - Once again the $\vec{B}$ field is affected by the presence of the material μ_r, but $\vec{H}$ is not
- Example: field inside a solenoid
  - Consider a very long solenoid with n turns per meter filled with a magnetic material with relative permeability of μ_r, with current I_0 through the wire; what is $\vec{H}, \vec{B}$ inside the solenoid?
  - $\vec{H}, \vec{B}$ will be in the same direction, based on RHR, let this be $\hat{a}_z$ so $\vec{B} = B_z \hat{a}_z$
  - Using Ampere's law, with a contour along the edge of the solenoid of length w that encloses the wire
  - When the solenoid is infinitely long, there is no magnetic field outside
  - Therefore $\oint_C \vec{B} \cdot \mathrm{d}\vec{l} = wB_z$ since only the piece of the contour inside the material gives a nonzero dot product
  - The enclosed current is $I_0 nw$
    - For n turns per meter, width of w, nw is the number of turns; therefore nwI_0 is the total current for all these loops
  - $wB_z = \mu_r \mu_0 nwI_0 \implies B = \mu_r \mu_0 n I_0 \hat{a}_z$
  - If we have N turns over L meters, then $\vec{B} = \frac{\mu_r \mu_0 I_0 N}{L} \hat{a}_z$
  - $\vec{H} = nI_0 \hat{a}_z = \frac{NI_0}{L} \hat{a}_z$

Ferromagnetic Materials

- On an atomic level, there are 2 major sources of magnetic dipoles:
  - Orbital motion of the electrons around the nucleus
    - This gives an orbital magnetic dipole moment m_0
  - Electron spin
    - This gives a spin magnetic dipole moment m_s
    - Thee two states of spin means that m_s is either parallel or antiparallel to the applied field
- Materials with non-zero internal moments can align ($\mu_r > 1$)
  - *Ferromagnetic* materials have their fields greatly enhanced (strong alignment) ($\mu_r \gg 1$)
  - *Paramagnetic* materials have their fields only slightly enhanced (weak alignment) $\mu_r \approx 1, \mu_r > 1$
  - *Ferrimagnetic* materials are in-between and have $\mu_r > 1$ but not too big; they're useful for higher frequency circuits (e.g. ferrites)
- Materials with zero internal moments actually reduces the net magnetic field ($\mu_r < 1$)
  - *Diamagnetic materials* will have a field in the opposite direction and get repelled by the applied field ($\mu_r \approx 1, \mu_r < 1$)
    - In superconducting materials there will be perfect diamagnetism (the field is perfectly canceled inside the material); this causes levitation (Meissner effect)

Hysteresis

- When a ferromagnetic material is magnetized, eventually it saturates and B begins to level off even with increasing H
- When the external field is turned off, B goes back down to B_r, the residual flux density – even though there's no more external field, the material stays magnetized
- At this point if we reverse the external current, we first reach the coercive H field or H_c, where the magnetization field disappears
  - At this point the permanent magnetization disappears
- If our applied field $\vec{H}$ varies with time (e.g. a sinusoidal AC current), we will go through the cycle of magnetization-demagnetization over and over

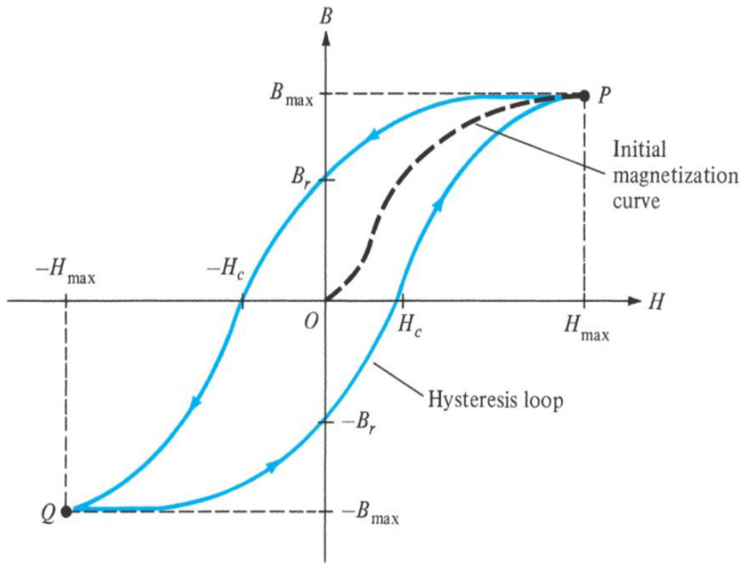

Figure 1: Hysteresis

  - This leads to significant energy losses, which we can show to be equal to the area of the hysteresis curve
- *Soft* magnetic materials have smaller B_r values and narrower hysteresis curves, while *hard* magnetic materials have larger B_r values and wider hysteresis curves
  - Soft materials are easily magnetized and demagnetized
  - Hard materials are difficult to demagnetize and make for good permanent magnets
  - The wider hysteresis curves of hard materials significantly increase the energy loss due to the magnetization-demagnetization cycles
- Since the relationship between $\vec{B}$ and $\vec{H}$ is no longer linear, for a ferromagnetic material we need to first determine its *operating condition* in order to determine its value of μ_r