Lecture 26, Mar 20, 2023

Magnetic Dipole

- A *magnetic dipole* is simply a closed loop of current, characterized by its *magnetic dipole moment* $\vec{m} = IS\hat{a}_n$, where the direction is determined through the right hand rule and S is the enclosed area
  - e.g. for a loop with radius a, we have $\vec{m} = \pi a^2 I\hat{a}_z$
  - If the loop has n turns, then the effective I is increased, so the magnetic dipole moment is magnified by a factor of n
  - $\vec{m} = nIS\hat{a}_n$ with units $[\mathrm{A\,m^2}]$
  - A magnetic dipole will produce a field in the same direction as the direction it points in
- What happens to a magnetic dipole moment in a $\vec{B}$ field?
  - The loop will experience some magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ (from $\vec{F}_m = q\vec{u} \times \vec{B}$)
  - This produces a net torque $\vec{T} = \vec{m} \times \vec{B}$
  - When $\vec{m}$ and $\vec{B}$ are aligned, the torque goes to zero; therefore a magnetic dipole will rotate until its own field is aligned with the applied field

Magnetization

- All materials have small atomic magnetic dipoles caused by the movement of electrons around the nuclei
  - Since they're all randomly oriented, there is no net field
- In a magnetic material, in the presence of an external magnetic field, the dipoles experience a torque that aligns them in the same direction as the field
- The overall result is that the small $\vec{B}$ fields from the dipoles now all point in the same direction, producing a net magnetic field
  - The magnetic field produced by the dipoles is in the same direction as the external applied field, so they add together
- A material is *magnetic* if it allows their atomic magnetic dipoles to be all aligned in the same direction
- Define the *magnetization vector* $\vec{M}$ (akin to $\vec{P}$'s relationship with $\vec{p}$) as an average of the magnetic dipoles within a material:
  - $\vec{M} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_i \vec{m}_i \approx N\vec{m}$ with units $[\mathrm{A/m}]$
- This magnetization leads to a *bound current density* (surface) $\vec{J}_{ms} = \vec{M} \times \hat{a}_n$ where $\hat{a}_n$ is the outward normal of the surface
  - There could also be volume bound current densities
- Now we can define 3 new quantities:
  - The magnetic field intensity $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu_r \mu_0}$
  - The magnetic susceptibility χ_m, where $\vec{M} = \chi_m \vec{H}$
  - The relative permeability $\mu_r = \chi_m + 1$
  - Like in the electric field case, $\vec{B}$ accounts for both bound and free currents, but $\vec{H}$ only cares about free currents

Important

The magnetic flux density in a magnetized material is not always greater than the applied field, since the magnetic dipole moments can also align to be antiparallel to the applied field, depending on the material (as a consequence, χ_m is not necessarily positive, so μ_r could be less than 1)

- Example: cylindrical permanent magnet, where a constant uniform $M = M_0\hat{a}_z$ exists; the cylinder is defined by $-\frac{L}{2} \le z \le \frac{L}{2}, 0 \le r \le a$
  - $\vec{J}_{ms} = \vec{M} \times \hat{a}_n = \vec{M} \times \hat{a}_r = M_0\hat{a}_\phi$

- $\vec{B} = \iint \frac{\mu_0 \vec{J}_{ms} \times (\vec{R} - \vec{R}')}{4\pi|\vec{R} - \vec{R}'|^3} a \, \mathrm{d}\phi' \, \mathrm{d}z'$

  $= \frac{\mu_0}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_0^{2\pi} \frac{M_0 \hat{a}_\phi \times (-a\hat{a}_r + (z - z')\hat{a}_z)}{(a^2 + (z - z^2)^2)^{\frac{3}{2}}} a \, \mathrm{d}\phi' \mathrm{d}z'$

  $= \frac{\mu_0 M_0}{2} \left(\frac{\frac{L}{2}}{\sqrt{a^2 + (z - L/2)^2}} + \frac{\frac{L}{2}}{\sqrt{a^2 + (z + L/2)^2}} \right) \hat{a}_z$
- What if $L \gg a$?
  * $\vec{B} \to \mu_0 M_0 \hat{a}_z$

Summary

When a magnetic material is exposed to an external applied magnetic field, it is magnetized; the magnetization is characterized by the magnetization vector,

$$\vec{M} = \chi_m \vec{H}$$

where the magnetic field intensity is defined as

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu_r \mu_0}$$

where the relative permeability is defined as

$$\mu_r = \chi_m + 1$$

The magnetization creates a surface bound current density,

$$\vec{J}_{ms} = \vec{M} \times \hat{a}_n$$

where $\hat{a}_n$ is the surface normal vector, and also a volume bound current density,

$$\vec{J}_m = \vec{\nabla} \times \vec{M}$$