Lecture 25, Mar 17, 2023

Ampere's Law

Definition

Ampere's Law in differential form is given by:

$$\vec{\nabla} \times \vec{H} = \vec{\nabla} \times \left(\frac{1}{\mu_r \mu_0} \vec{B}\right) = \vec{J}$$

Where the magnetic field intensity $\vec{H}$ is related to the magnetic flux density $\vec{B}$ as

$$\vec{B} = \mu_r \mu_0 \vec{H}$$

In integral form, this is

$$\oint_C \vec{H} \cdot \mathrm{d}\vec{l} = \iint_S \vec{J} \cdot \mathrm{d}\vec{s} = I_{enc}$$

- Ampere's law is a fundamental law, the analogue of Gauss's law
- At every point in space, the magnetic field intensity has a nonzero curl only if a current density $\vec{J}$ is present at that point
- The integral form tells us that if we take any contour integral of $\hat{H}$, it is equal to the current crossing through the surface enclosed by that curve
    - Note that the direction of $\mathrm{d}\vec{s}$ in relation to C is defined based on the right hand rule (coming from Stokes' theorem), which is what gives us the right hand rule for $\vec{B}$
- To find $\vec{H}$ from $\vec{J}$ is like finding $\vec{E}$ using Gauss's law; instead of using a Gaussian surface, we use an Amperian loop
    - Choose the loop so that:
        * $\vec{H}$ is always tangential or normal to the loop
        * $\vec{H}$ has a constant value where $\vec{H}$ is a tangential
    - This means $\displaystyle\int \vec{H} \cdot \mathrm{d}\vec{l} = \int H \,\mathrm{d}l = HL$ where L is the length of the loop where $\hat{H}$ is tangential to the loop
- For an infinitely long wire in the $\vec{a}_z$ direction, we choose the Amperian loop to be a circle centered on the wire, which gets us $\displaystyle\oint_C \vec{H} \cdot \mathrm{d}\vec{l} = 2\pi r H_\phi = I_{enc} \implies \vec{H} = \frac{I_0}{2\pi r}\hat{a}_\phi$