Lecture 21, Mar 6, 2023

Electric Field Inside Conducting Materials

- When an electric field is applied to a material where there are free charge carriers, it will create a current density $\vec{J}$
  - Electrons move with a drift velocity $\vec{u}_d$
- The force on each conductor is $\vec{F}_e = -e\vec{E} = m_e\vec{a} \implies \vec{a} = -\frac{e}{m_e}\vec{E}$
- The charge density is $\rho_{ve} = -N_e e$ where N_e is the charge carrier density; so define the current density as $\vec{J} = \rho_{ve}\vec{u}_d$
  - Current density has units of $\mathrm{A/m^2}$ which this relation satisfies
- By convention current density $\vec{J}$ is in the same direction as the electric field $\vec{E}$
  - $\vec{u}_d$ would be in the opposite direction as $\vec{E}$ and $\vec{J}$ if the charge carriers were electrons
- How can we model the movement of electrons?
  - Consider the case where there is no $\vec{E}$ field applied, electrons move by thermal agitation and bounce around atoms
    * There is no net movement since the movements are completely random
    * The velocity is on the order of 1×10^5 m/s but there is no coordination in direction, so no net movement
  - When a field is applied, there is a net movement in the direction that the field pushes the electrons in
    * The overall average velocity is the drift velocity $\vec{u}_d = \Delta t\vec{a} \approx \tau\vec{a} = -\frac{\tau e\vec{E}}{m_e}$
    * τ is the mean free time, or average time between collisions
  - Since $\vec{u}_d$ is directly connected to current density, higher τ means better conductor
  - Define the mobility $\mu_e = -\frac{e\tau}{m_e}$ so that $\vec{u}_d = \mu_e\vec{E}$
    * The mobility takes into account both τ and the type of charge carrier
- Since the current density is current per unit area, $I = \iint_S \vec{J} \cdot \mathrm{d}\vec{S}$
- Therefore $\vec{J} = \rho_{ve}\vec{u} = -N_e e\left(-\frac{\tau e}{me}\right) = \frac{N_e e^2\tau}{m_e}\vec{E} = \sigma\vec{E}$

Definition

Ohm's Law in point form:

$$\vec{J} = \sigma\vec{E}$$

where $\sigma = \frac{N_e e^2 \tau}{m_e}$ is the conductivity of the material

- Using this we can derive another equation for the boundary condition: $E_{t1} = E_{t2} \implies \frac{J_{t1}}{\sigma_1} = \frac{J_{t2}}{\sigma_2}$
  - Combine this to with the boundary condition for $\vec{D}$ we have $\frac{\varepsilon_{r1}\varepsilon_0 J_{n1}}{\sigma_1} - \frac{\varepsilon_{r2}\varepsilon_0 J_{n2}}{\sigma_2} = \rho_s$
  - For a steady current interface, J is continuous: $J_{n1} = J_{n2} = J_n$, therefore $\rho_s = J_n\left(\frac{\varepsilon_{r1}\varepsilon_0}{\sigma_1} - \frac{\varepsilon_{r2}\varepsilon_0}{\sigma_2}\right)$

Summary

Electric current quantities:

- N_e charge carrier density (number density of charge carriers)
- $\rho_{ve} = -N_e e$ charge density (density of moving charges)
- $\vec{J} = \rho_{ve}\vec{u}_d$ current density (current per unit area)
- $\vec{u}_d = \mu_e \vec{E} = -\frac{\tau e \vec{E}}{m_e}$ drift velocity (average velocity of moving electrons)
- $\mu_e = -\frac{e\tau}{m_e}$ (electron) mobility (how easily electrons move given an applied electric field)
- $\sigma = \frac{N_e e^2 \tau}{m_e}$ conductivity

Summary

Boundary conditions for current density for a current going from material 2 to material 1:

- Tangential component: $\frac{J_{t1}}{\sigma_1} = \frac{J_{t2}}{\sigma_2}$
- Normal component: $\frac{\varepsilon_{r1}\varepsilon_0 J_{n1}}{\sigma_1} - \frac{\varepsilon_{r2}\varepsilon_0 J_{n2}}{\sigma_2} = \rho_s$
- Given a steady current interface, we can find $\rho_s = J_n \left(\frac{\varepsilon_{r1}\varepsilon_0}{\sigma_1} - \frac{\varepsilon_{r2}\varepsilon_0}{\sigma_2} \right)$