Lecture 2, Jan 11, 2023

The Electric Field

- Expressing everything in terms of forces is cumbersome; using "fields" can simplify this
- While forces can be directly experienced, fields remove the immediate effects
  - An electric force requires two object, a source charge (the thing causing the force), and a test charge (the thing experiencing the force)
  - A field however only requires the source charge and fully characterizes it
- We can define the *electric field intensity* as $\vec{E}_{12} = \lim_{Q_2 \to 0} \frac{\vec{F}_{12}}{Q_{12}} = \lim_{Q_2 \to 0} \frac{1}{Q_{12}} k \frac{Q_1 Q_2}{R^2} \hat{a}_{12} = k \frac{Q_1}{R^2} \hat{a}_{12}$
  - The electric field has units of [N/C] = [V/m]
  - Now we have $\vec{F}_{12} = Q_2 \vec{E}_{12}$

Definition

The *electric field* caused by a charge Q_1 is defined as

$$\vec{E}_{12} = k \frac{Q_1}{R^2} \hat{a}_{12} = \frac{Q_1}{4\pi\varepsilon_0 R^2} \hat{a}_{12}$$

The electric field has 4 key properties:

1. $\vec{E}$ points away from positive charges
2. $\vec{E}$ points towards negative charges
3. $\vec{E}$ points along the line connecting the source point to the measurement point
4. $\vec{E}$ is linear, so we can superimpose electric fields from multiple charges

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The electric field at point $\vec{R}$ due to a point charge at $\vec{R}'$ is

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_0 \|\vec{R} - \vec{R}'\|^2} \hat{a}_{12} = \frac{Q_1}{4\pi\varepsilon_0 \|\vec{R} - \vec{R}'\|^3} (\vec{R} - \vec{R}')$$

- The first 2 properties are by convention – we always think of the electric field coming from a positive charge and going into a negative charge somewhere

Position Vectors

- In Cartesian coordinates, a point $P(x, y, z)$ is specified by a position vector $\vec{R} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ where $\hat{a}_x, \hat{a}_y, \hat{a}_z$ are the 3 unit vectors
- In cylindrical coordinates, a point is specified by $P(r, \phi, z)$; unit vectors are $\hat{a}_r, \hat{a}_\phi, \hat{a}_z$
  - $\hat{a}_z$ is constant, but $\hat{a}_r, \hat{a}_\phi$ change based on angle!
  - A position vector is described by $\vec{R} = r\hat{a}_r + z\hat{a}_z$, because ϕ is encoded in $\hat{a}_r$
    - In Cartesian coordinates $\hat{a}_r = \cos(\phi)\hat{a}_x + \sin(\phi)\hat{a}_y$
- In spherical coordinates, a point is specified by $P(R, \theta, \phi)$ (**note ϕ is the angle in the xy plane!**); unit vectors are $\hat{a}_R, \hat{a}_\phi, \hat{a}_\theta$
  - All unit vectors change based on where you are
  - A position vector is described by $\vec{R} = R\hat{a}_r$
  - $\hat{a}_r = \sin\theta\cos\phi\hat{a}_x + \sin\theta\sin\phi\hat{a}_y + \cos\theta\hat{a}_z$