Lecture 19, Mar 1, 2023

Boundary Value Problems

- Motivation: usually we don't have any idea what the charge distribution ρ_s is like
- We often know what values of V are on the boundaries of the problem
- By using Laplace's or Poisson's equations we can determine $\vec{E}$ in a given problem without knowing the charge densities
- Example: parallel plate capacitor
  - Assume $\rho_v = 0$ and ε_r is constant, so we use Laplace's equation $\vec{\nabla}^2 V = 0$
  - Assume $\vec{E} = E\hat{a}_x$, then $\vec{\nabla}^2 \vec{V} = \dfrac{\mathrm{d}^2 V}{\mathrm{d}x^2} = 0 \implies V(x) = c_1 x + c_2$
  - Using boundary conditions $V(0) = V_0, V(d) = 0$ we get $V(x) = -\dfrac{V_0}{d}x + V_0$
- In general, start with Poisson's equation; if the field is homogeneous we can take out ε; if there is no charge density then we can use Laplace's equation
- Then use the equation to double integrate to find V, using boundary conditions to find the constants, then find $\vec{E}$
- Finally from E we may find other quantities such as Q with a variety of methods (Gauss's Law, boundary conditions, finding work, etc)