

Lecture 18, Feb 27, 2023

Laplace's and Poisson's Equations

- Since $\vec{\nabla} \cdot \vec{D} = \rho_v$, we have $\vec{\nabla} \cdot (\epsilon \vec{E}) = \rho_v$
- $\vec{\nabla} \times \vec{E} = 0 \implies \vec{E} = -\vec{\nabla}V$
- Combining these we get Poisson's equation: $\vec{\nabla} \cdot (\epsilon \vec{\nabla}V) = -\rho_v$
- In the case where $\rho_v = 0$ we get Laplace's equation: $\vec{\nabla} \cdot (\epsilon \vec{\nabla}V) = 0$
- If the electric field is homogeneous, then we may take out ϵ and get $\vec{\nabla} \cdot \vec{\nabla}V = -\frac{\rho_v}{\epsilon} \implies \vec{\nabla}^2V = -\frac{\rho_v}{\epsilon}$

Equation

Poisson's equation:

$$\vec{\nabla} \cdot (\epsilon \vec{\nabla}V) = -\rho_v$$

In a homogeneous material, this reduces to

$$\vec{\nabla}^2V = -\frac{\rho_v}{\epsilon}$$

Where there is no charge density, these are called Laplace's equations