Lecture 14, Feb 10, 2022

Dielectric Breakdown

- When a strong enough electric field is applied, even in a dielectric the electrons can jump from the valence to the conduction band
  - When this happens, the material becomes a conductor; this is referred to as *dielectric breakdown*
  - The field is strong enough to overcome the attractive force between the nucleus and its orbiting electrons; the atom goes beyond just stretching and the electrons are detached
- The *dielectric strength* E_b is the maximum electric field that the material can withstand before a current flows
  - E_B for air is 3×10^6 V/m
  - E_B for mica is 200×10^6 V/m
    - This is why mica is used for capacitors – the very small inter-plate distance means the same voltage creates a much larger electric field
- Lightning is a great example of this
  - Lightning rods work by concentrating the electric field at its end

Boundary Conditions for the Electric Field in Materials

- Application example: optical fibres
  - Low conductivity of the glass reduces conductive power loss
  - e.g. copper wire requires signal boosters every 10 km; with optical fibres boosters are only needed every 100 km or 1000 km
  - Optical fibres consists of an outer cylinder (cladding) with an index of refraction on the order of 1.2, and a core cylinder with an index of refraction slightly larger
    - Index of refraction is directly related to ε_r
  - A light source shines into the core, and most of the light is reflected and travels down the core
    - When the light hits the interface between the cladding and the core, total internal reflection happens
  - Total internal reflection happens due to the boundary conditions
  - The fibre is a *waveguide* that carries electromagnetic waves
- Consider the boundary of two materials 1 and 2, to see how an electric field behaves at the boundary we apply Maxwell's equations
- Break the electric field into tangential and normal components; these components are affected differently, and based on how they are affected the field lines bend
- Consider the tangential components $\vec{E}_{t1}, \vec{E}_{t2}$
  - From Faraday's law $\oint_C \vec{E} \cdot \mathrm{d}\vec{l} = 0$
  - We can create a contour right on the boundary with infinitesimal thickness, so we can isolate the boundary tangential components
  - $\oint_C \vec{E} \cdot \mathrm{d}\vec{l} = E_{t2}\Delta l - E_{t1}\Delta l = 0 \implies E_{t1} = E_{t2} \implies \frac{D_{t1}}{\varepsilon_{r1}} = \frac{D_{t2}}{\varepsilon_{r2}}$
  - At the boundary of an interface, tangential components of the field have to be the same
- Consider the normal components $\vec{D}_{n1}, \vec{D}_{n2}$
  - From Gauss's law $\oiint_S \vec{D} \cdot \mathrm{d}\vec{S} = Q_{enc}$
  - Use a Gaussian cylinder, with parts just above and just below the boundary, with surface area ΔS
  - Through this cylinder we have D_{n2} going in from the bottom and D_{n1} coming out from the top
  - In the limit: $\oiint_S \vec{D} \cdot \mathrm{d}\vec{S} = -D_{n2}\Delta S + D_{n1}\Delta s = \rho_s \Delta S \implies D_{n1} - D_{n2} = \rho_s$ or $\hat{n}_2 \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$
  - This gives us $\varepsilon_{r1}\varepsilon_0 E_{n1} - \varepsilon_{r2}\varepsilon_0 E_{n2} = \rho_s$
  - When there is no surface free charge, $\varepsilon_{r1} E_{n1} = \varepsilon_{r2} E_{n2}$

Summary

At the boundary between two dielectrics, taking the normal direction to be pointing from material 2 to material 1:

$$E_{t1} = E_{t2} \implies \frac{D_{t1}}{\varepsilon_{r1}} = \frac{D_{t2}}{\varepsilon_{r2}}$$

$$D_{n1} - D_{n2} = \rho_s \implies \varepsilon_{r1}\varepsilon_0 E_{n1} - \varepsilon_{r2}\varepsilon_0 E_{n2} = \rho_s$$

or when there is no free charge at the boundary:

$$D_{n1} = D_{n2} \implies \varepsilon_{r1} E_{n1} = \varepsilon_{r2} E_{n2}$$