Lecture 12, Feb 6, 2023

Polarization

- Consider the effect of a static field on the atoms in an insulator:
  - The field inside the insulator, $\vec{E}_0$, *polarizes* the bond atoms within the material – the atoms are stretched, negative charges and positive charges are pushed to different sides
  - The insulator has become a *dielectric* (refers to the separation of the positive and negative charges)
  - This separation of charge creates an electric field of its own, the *polarization field* $\vec{E}_n$
    - Now the total field is reduced: $\vec{E}_{tot} = \vec{E}_0 - \vec{E}_p$
  - The polarized atoms can be approximated with an *electric dipole*
- The dipole is represented by the *dipole moment* $\vec{p} = Q\vec{d}$, where $\vec{d}$ is the vector connecting the two charges with $-Q$ and $+Q$
  - $\vec{d}$ and thus $\vec{p}$ always points from the negative charge to the positive charge by convention

Definition

The *dipole moment* of a pair of charges $-Q$ and Q, where $\vec{d}$ is the vector pointing from the negative charge to the positive charge, is

$$\vec{p} = Q\vec{d}$$

The *polarization vector* of a material is

$$\vec{P} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{i=1}^{N\Delta v} \vec{p}_i \approx N\vec{p}_i \qquad \text{if } \vec{p}_i \text{ are the same}$$

where N is the number of atoms per unit volume

- The polarization vector has units of $\mathrm{C/m^2}$; it is a measure of the "average" dipole moment per unit volume
- In this course we are concerned with *simple media*, that is:
  - Linear: The properties of the dielectric do not depend on the electric field strength
    - The strength of the polarization is directly proportional to the applied field
  - Isotropic: The properties of the dielectric do not depend on the field direction
    - The direction of polarization is always parallel to the direction of the applied field
  - Homogeneous: The properties of the dielectric do not depend on the electric field position (i.e. there's only one type of material)
    - The relationship between the polarization and the electric field is the same everywhere within the material
- For such simple media, $\vec{P} = \varepsilon_0 \chi_e \vec{E}$
  - χ_e is the *electric susceptibility* of the material, a unitless quantity – higher χ_e means higher polarization
    - χ_e can be a matrix if the material is not isotropic
  - Define $\varepsilon_r = \chi_e + 1$ so that $\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}$ where ε_r is the *relative permittivity* of the material
    - Therefore $\vec{D} = \varepsilon\vec{E} = \varepsilon_r\varepsilon_0\vec{E}$
- At the surfaces of the material, there's no more atoms, we have a negative charge layer and a positive charge layer, which causes attraction
  - This results in the *bound charge density* $\pm\rho_{sb}$ (as opposed to the free charge densities $\pm\rho_s$)
  - Therefore $\vec{E}_p = \dfrac{\rho_{sb}}{\varepsilon_0}$ or $\|\vec{E}_{tot}\| = \|\vec{E}_0\| - \|\vec{E}_p\| = \dfrac{1}{\varepsilon_0}(\rho_s - \rho_{sb})$
  - **This is equivalent to reducing it by a factor of ε_r: $\vec{E}_{tot} = \dfrac{\vec{E}_0}{\varepsilon_r}$**
- In a polarized material:
  - $\vec{E}_0$ is the original field applied, which results in free charge densities

    - Bound charge densities $\rho_{sb} = \vec{P} \cdot \vec{a}_n$ where $\vec{a}_n$ are the outward normal vectors of the surface
        * This gives $\rho_{sb} = \|\vec{P}\|$ if the polarization vector is normal to the surface
- When dealing with these problems, it's important to note whether the potential difference or the ρ_s are constant

Summary

To account for polarization in a material, all we need to do is to add the relative permittivity ε_r:

$$\vec{E} = \frac{\vec{E}_0}{\varepsilon_r}$$

The polarization induces bound charges, with surface bound charge density given by

$$\rho_{sb} = \vec{P} \cdot \hat{a}_n$$

where $\hat{a}_n$ is the unit normal of the surface; the volume bound charge density is given by

$$\rho_{vb} = -\vec{\nabla} \cdot \vec{P}$$

$\vec{P}$ is the polarization vector, given by

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} = \varepsilon_0(\varepsilon_r - 1)\vec{E} = \vec{D} - \varepsilon_0 \vec{E}$$