Lecture 10, Feb 1, 2023

Relating the Electric Field and Electric Potential

- Because $\vec{\nabla} \times \vec{E} = 0$ for electrostatic fields (Faraday's Law), it is the gradient of a scalar field
  - This also means that moving in a closed loop does not do any work
- We let $\vec{E} = -\vec{\nabla}\vec{V}$ where $\vec{V}$ is the electric scalar potential
  - The negative sign is to make sure the electric field points away from positive charges
- Note $\vec{E}$ only cares about the spatial rate of change of V, not its actual value, so it doesn't matter if the potential is absolute or relative

Gradient

- The gradient is the direction and magnitude of the maximum spacial rate of change
- The gradient in different coordinate systems is given by:
  - $\dfrac{\partial V}{\partial x}\hat{a}_x + \dfrac{\partial V}{\partial y}\hat{a}_y + \dfrac{\partial V}{\partial z}\hat{a}_z$
  - $\dfrac{\partial V}{\partial r}\hat{a}_r + \dfrac{1}{r}\dfrac{\partial V}{\partial \phi}\hat{a}_\phi + \dfrac{\partial V}{\partial z}\hat{a}_z$
  - $\dfrac{\partial V}{\partial R}\hat{a}_R + \dfrac{1}{R}\dfrac{\partial V}{\partial \theta}\hat{a}_\theta + \dfrac{1}{R\sin\theta}\dfrac{\partial V}{\partial \phi}\hat{a}_\phi$
- The factors of $\dfrac{1}{r}$ etc can be thought of as making sure the gradient has the right units