Lecture 1, Jan 9, 2023

Maxwell's Equations

Equation

Maxwell's Equations:

$$\text{Faraday's Law: } \vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\text{Ampere's Law: } \vec{\nabla} \times \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\text{Gauss's Law (Electric): } \vec{\nabla} \cdot \vec{E} = \frac{\rho_\nu}{\varepsilon}$$

$$\text{Gauss's Law (Magnetic): } \vec{\nabla} \cdot \vec{H} = 0$$

In a static field:

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_\nu}{\varepsilon}$$

$$\vec{\nabla} \times \vec{H} = 0$$

- In a static field, electric and magnetic fields are now independent
- Maxwell added the second term in Ampere's Law, connecting electric and magnetic fields

Electrostatics – The Beginning

- The *triboelectric series* ranks the tendency for different materials to gain or lose electrons
- Coulomb noticed the properties of the electric force (Coulomb's Law)
  - $|\vec{F}_e|$ dependent on $Q_1 Q_2$
  - $|\vec{F}_e| \propto \frac{1}{R^2}$
  - The direction of $\vec{F}_e$ acts along the line connecting Q_1 and Q_2
  - Like charges repel, opposite charges attract
- Mathematically we express this as $|\vec{F}_e| = F_e \propto \frac{Q_1 Q_2}{R^2}$

Definition

Coulomb's Law (scalar form):

$$F_e = k\frac{Q_1 Q_2}{R^2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

where $k = 9 \times 10^9\,\mathrm{Nm^2/C^2}$, $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F/m}$ is the permittivity of free space