Lecture 23, Dec 5, 2023

Exam Review

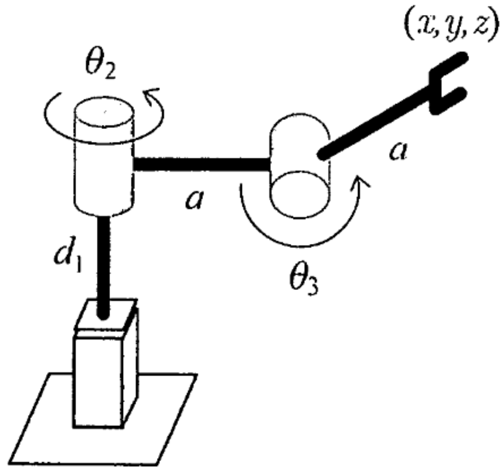

Figure 1: Example question 1.

- Refer to the manipulator example from Lecture 19
- What is the manipulability index of this manipulator, based on the volume of the manipulability ellipsoid?
  - We need to find $w = \sigma_1\sigma_2\sigma_3$, where σ_i are the singular values of $\boldsymbol{J}^{(v)}$
  - Since σ_i^2 are the eigenvalues of $\boldsymbol{J}\boldsymbol{J}^T$, $w = \sqrt{\lambda_1\lambda_2\lambda_3} = \sqrt{\det(\boldsymbol{J}\boldsymbol{J}^T)} = |\det \boldsymbol{J}|$, since $\boldsymbol{J}$ is square
  - Recall from Lecture 19: $|\det \boldsymbol{J}| = a^2|\sin\theta_3|(1 + \cos\theta_3)$

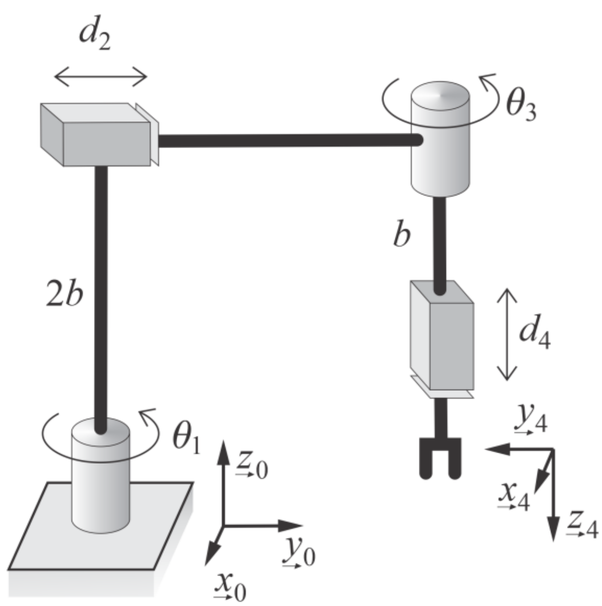

Figure 2: Example question 2.

- Consider the RPRP manipulator above (note the direction of θ_3) is reversed

- Determine the DH transformation in $SE(3)$, $\boldsymbol{T}_{04}$
  - $\alpha_1 = 0, \alpha_2 = 270°, \alpha_3 = 270°, \alpha_4 = 0$
  - $\boldsymbol{T}_{01} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
    - This is a rotation about the $\underset{\rightarrow}{z}_1$ axis by an angle θ_1
  - $\boldsymbol{T}_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 2b \\ 0 & 0 & 0 & 1 \end{bmatrix}$
    - This is a rotation plus a translation
  - $\boldsymbol{T}_{23} = \begin{bmatrix} c_3 & 0 & -s_3 & 0 \\ 0 & 0 & 1 & 0 \\ -s_3 & -c_3 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
  - $\boldsymbol{T}_{34} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & b \\ 0 & 0 & 0 & 1 \end{bmatrix}$
  - $\boldsymbol{T}_{04} = \begin{bmatrix} c_{13} & s_{13} & 0 & -d_2 s_1 \\ s_{13} & -c_{13} & 0 & d_2 c_1 \\ 0 & 0 & -1 & b \\ 0 & 0 & 0 & 1 \end{bmatrix}$
  - Note this only takes us to link 4, but not the position of the end-effector since we need an additional length d_4
- Considering the pose of the end-effector to be only (x, y, z, ϕ), where ϕ is the rotation angle about the vertical axis, determine the theoretical singularities of the manipulator, if any
  - We can determine the manipulator pose in terms of joint variables by inspection
  - $\begin{bmatrix} x \\ y \\ z \\ \phi \end{bmatrix} = \begin{bmatrix} -d_2 \sin\theta_1 \\ d_2 \cos\theta_1 \\ b - d_4 \\ \theta_1 - \theta_3 \end{bmatrix}$
    - Note when $\theta_1 = 0$ the prismatic joint is aligned with $\underset{\rightarrow}{y}_0$
  - $\boldsymbol{J} = \begin{bmatrix} -d_2 \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ -d_2 \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$
  - Singularities occur when $\det(\boldsymbol{J}\boldsymbol{J}^T) = 0 \iff \det \boldsymbol{J} = 0$
  - $\det \boldsymbol{J} = -(-1)(-1)(-d_2 \cos^2\theta_1 - d_2 \sin^2\theta_1) = d_2$
  - Hence the singularity is at $d_2 = 0$ – however this is theoretical, since for a real manipulator we can never collapse d_2 to exactly 0
- If there's only a force f_y^{ee} acting in the y direction (in the global frame) at the end effector, what must the joint control forces and torques be to balance it?
  - $\boldsymbol{\eta} = \begin{bmatrix} \tau_1 \\ f_2 \\ \tau_3 \\ f_4 \end{bmatrix} = \boldsymbol{J}^T \boldsymbol{f}^{ee} = \boldsymbol{J}^T \begin{bmatrix} f_x \\ f_y \\ f_z \\ \tau_z \end{bmatrix} = \boldsymbol{J}^T \begin{bmatrix} 0 \\ f_y^{ee} \\ 0 \\ 0 \end{bmatrix}$
  - We need to be careful here since we're working in 4-dimensional space instead of 6-dimensional space
  - Therefore $\boldsymbol{\eta} = f_y^{ee} \begin{bmatrix} -d_2 \sin\theta_1 \\ \cos\theta_1 \\ 0 \\ 0 \end{bmatrix}$