Lecture 19, Nov 21, 2023

Manipulator Examples

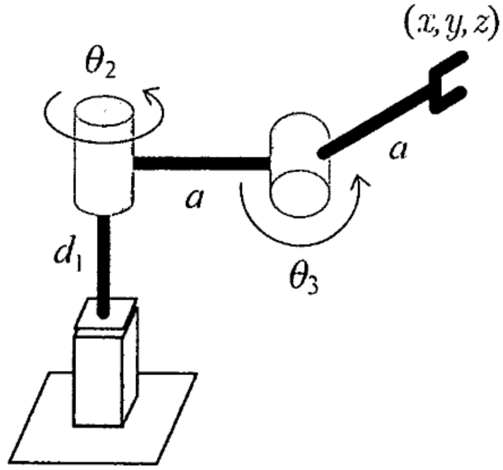

Figure 1: Example question.

- For the PRR manipulator above, with joint variables d_1, θ_2, θ_3 (where θ_3 is measured relative to the second link), determine:
  - Displacement (x, y, z) of the end-effector
    * We can do this by inspection from the geometry
    * $\boldsymbol{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a(1+\cos\theta_3)\cos\theta_2 \\ a(1+\cos\theta_3)\sin\theta_2 \\ d_1 + a\sin\theta_3 \end{bmatrix}$
  - The Jacobian $\boldsymbol{J}^{(v)}$, with the translational velocity only
    * Since we have $\boldsymbol{r}(d_1, \theta_2, \theta_3)$ we can directly differentiate to find the Jacobian
    * $\boldsymbol{J}^{(v)} = \begin{bmatrix} 0 & -a(1+\cos\theta_3)\sin\theta_2 & -a\sin\theta_3\cos\theta_2 \\ 0 & a(1+\cos\theta_3)\cos\theta_2 & -a\sin\theta_3\sin\theta_2 \\ 1 & 0 & a\cos\theta_3 \end{bmatrix}$
  - Singularities of the system
    * The singularity condition is $\det(\boldsymbol{J}\boldsymbol{J}^T) = 0$, which for a square $\boldsymbol{J}$ is equivalent to $\det \boldsymbol{J} = 0$
    * Since the first column has only a single 1, the determinant is given by the determinant of the 2x2 matrix at the top right
    * $$\begin{aligned} \det \boldsymbol{J} &= a(1+\cos\theta_3)\sin\theta_2 a\sin\theta_3\sin\theta_2 + a\sin\theta_3\cos\theta_2 a(1+\cos\theta_3)\cos\theta_2 \\ &= a^2(1+\cos\theta_3)\sin\theta_3\sin^2\theta_2 + a^2(1+\cos\theta_3)\sin\theta_3\cos^2\theta_2 \\ &= a^2\sin\theta_3(1+\cos\theta_3) \end{aligned}$$
    * This gives us $\theta_3 = 0, \pi$
    * Intuitively, at $\theta_3 = 0$, the last 2 links are aligned; this means we need an infinite $\dot{\theta}_3$ to get a finite EE velocity; at $\theta_3 = \pi$ the last links are folded on each other, so any angle of θ_2 results in the same EE position, and we also have the infinite velocity issue; additionally $\dot{d}_1$ results in the same $\dot{z}$ as $\dot{\theta}_3$
      - Notice $\theta_3 = \pi$ is a double root, which corresponds to the two different interpretations
  - The required joint force and torques required to deliver a force f^{ee} applied in the downward direction, when $d_1 = a, \theta_2 = 0, \theta_3 = -45°$
    * Recall: $\boldsymbol{\eta} = \boldsymbol{J}^T\boldsymbol{f}$

* In this configuration, $\boldsymbol{J} = \begin{bmatrix} 0 & 0 & \frac{a\sqrt{2}}{2} \\ 0 & a\left(1+\frac{\sqrt{2}}{2}\right) & 0 \\ 1 & 0 & a\frac{\sqrt{2}}{2} \end{bmatrix}$
* $\boldsymbol{f} = \begin{bmatrix} 0 \\ 0 \\ -f^{ee} \end{bmatrix}$
* Therefore $\boldsymbol{\eta} = \begin{bmatrix} -f^{ee} \\ 0 \\ -\frac{a\sqrt{2}}{2} f^{ee} \end{bmatrix}$