Lecture 17, Nov 14, 2023

Manipulators

- Two types of topologies: *open chains*, where there are no closed loops in the system, and *closed chains*
- We consider 2 types of joints: *revolute* (i.e. rotational) and *prismatic* (i.e. translational, extending/contracting)
  - Consider only 1 degree of freedom per joint

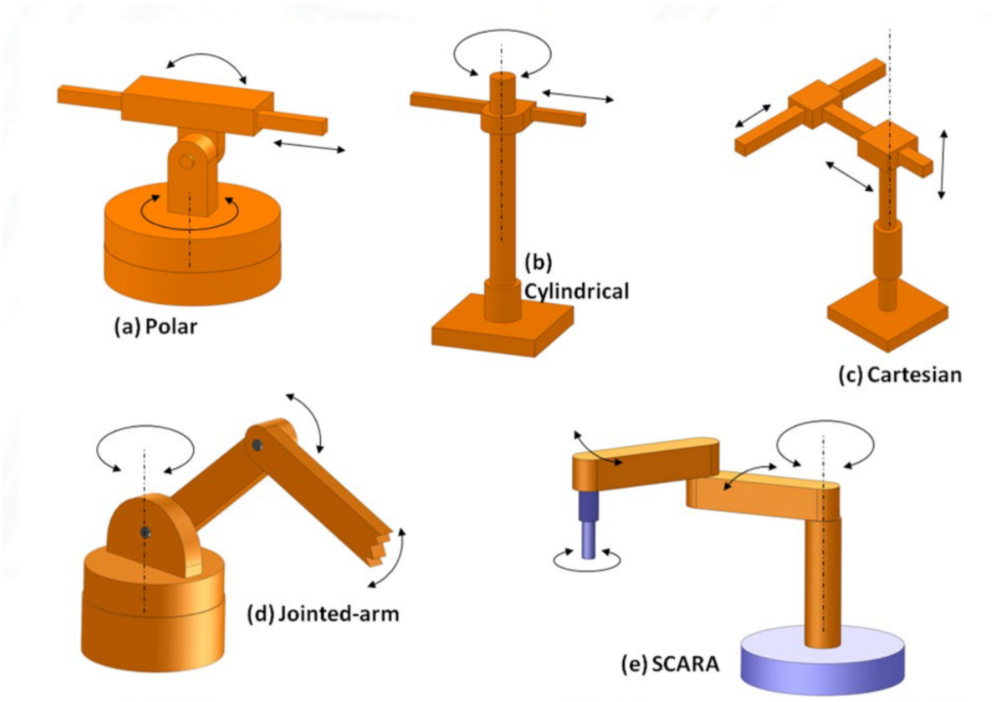

Figure 1: Types of manipulators.

- We are particularly interested in 3-DoF manipulators, because 3 independent degrees of freedom lets us place the end-effector anywhere in 3D translational space
  - Attaching another 3 degrees of freedom via a wrist will get us the rotations as well
- 3-DoF manipulators:
  - Cartesian: PPP (prismatic-prismatic-prismatic)
    * e.g. a 3D printer
    * Each degree of freedom covers a Cartesian coordinate
    * Workspace shape is a cube
  - Revolute/anthropomorphic: RRR (revolute-revolute-revolute)
    * e.g. ABB IRB1400
    * The joints are referred to as body, shoulder, forearm
  - SCARA (Selective Compliant Articulated Robot for Assembly): RRP (revolute-revolute-prismatic)
    * e.g. Epson E2L653S
  - Spherical/polar: RRP (revolute-revolute-prismatic)
    * e.g. the Stanford arm
    * Unlike SCARA the second revolute joint is rotated
  - Cylindrical: RPP (revolute-prismatic-prismatic)
    * e.g. Seiko RT3300

Manipulator Geometry

- Rotation matrices form the *special orthogonal group*, $SO(3) = \left\{ \boldsymbol{C} \in \mathbb{R}^{3\times3} \,\middle|\, \boldsymbol{C}^T\boldsymbol{C} = \mathbf{1}, \det \boldsymbol{C} = 1 \right\}$
  - Recall that a group is a set of elements G and a binary operation xy that is closed, associative, has an identity and inverse
  - Commutative groups (aka Abelian groups) have a commutative binary operation ($SO(3)$ is not Abelian)
  - $SO(3)$ is a *Lie group*, i.e. it is differentiable
- Given a point $\underrightarrow{w}$ expressed in $\underrightarrow{\mathcal{F}}_b$ relative to O_b, we may want to express it in $\underrightarrow{\mathcal{F}}_a$ relative to O_a; O_b has position $\underrightarrow{\rho}$ relative to O_a
  - $\underrightarrow{v} = \underrightarrow{w} + \underrightarrow{\rho} \iff \boldsymbol{v}_a = \boldsymbol{C}_{ab}\boldsymbol{w}_b + \boldsymbol{\rho}_a$
  - We can combine this as $\begin{bmatrix} \boldsymbol{v}_a \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_{ab} & \boldsymbol{\rho}_a \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{w}_b \\ 1 \end{bmatrix} \implies \boldsymbol{u}_a = \boldsymbol{T}_{ab}\boldsymbol{u}_b$
    * Note this only works for position vectors
  - $\boldsymbol{T}_{ab}$ is a 4×4 matrix that generalizes rotations
  - $\boldsymbol{T}$ forms the *special Euclidean group* $SE(3) = \left\{ \boldsymbol{T} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{\rho} \\ \mathbf{0}^T & 1 \end{bmatrix} \in \mathbb{R}^{4\times4} \,\middle|\, \boldsymbol{C}^T\boldsymbol{C} = \mathbf{1}, \det \boldsymbol{C} = 1 \right\}$
    * This is also a Lie group but not a commutative group
  - Note $\boldsymbol{T}^{-1} = \begin{bmatrix} \boldsymbol{C}^T & -\boldsymbol{C}^T\boldsymbol{\rho} \\ \mathbf{0}^T & 1 \end{bmatrix}$, and the identity of $SE(3)$ is $\mathbf{1}_{4\times4}$
  - In $SE(3)$, $\dot{\boldsymbol{T}}_{ab} = -\boldsymbol{\Omega}_a^{ab}\boldsymbol{T}_{ab}$ where $\boldsymbol{\Omega}_a^{ab} = \begin{bmatrix} \boldsymbol{\omega}_a^{ab\times} & \boldsymbol{v}_a^{ab} \\ \mathbf{0}^T & 0 \end{bmatrix}$
    * This is a generalized form of Poisson's kinematical equation

Denavit-Hartenberg Parameters

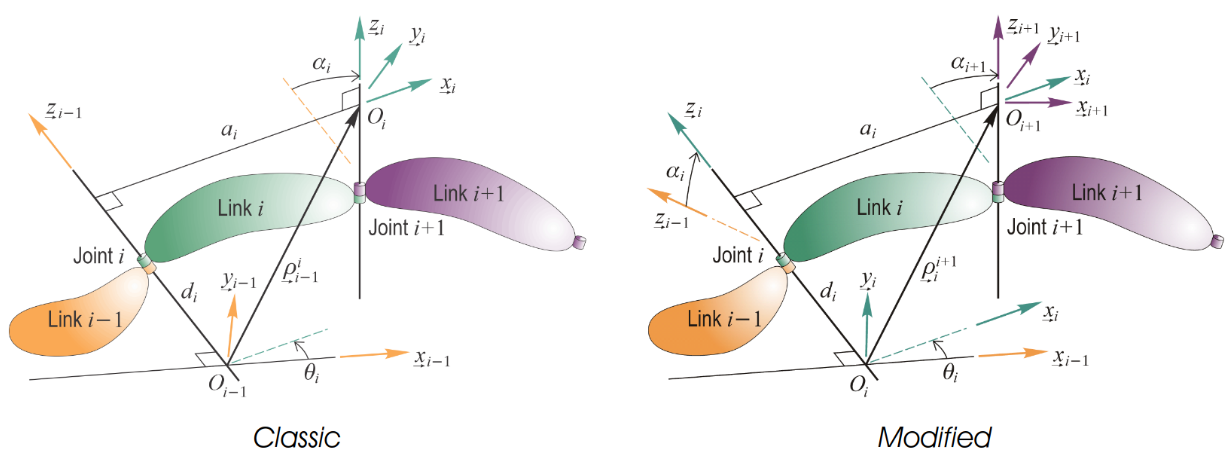

Figure 2: Denavit-Hartenberg Parameters.

- We can describe any series link manipulator with revolute and prismatic joints using *Denavit-Hartenberg parameters*
  - The DH parameters consist of 4 parameters per joint:
    1. Link length (a_i)
       * This is the length of a line segment normal to and joining $\underrightarrow{z}_i, \underrightarrow{z}_{i+1}$ (direction $\underrightarrow{x}_i = \underrightarrow{z}_i \times \underrightarrow{z}_{i+1}$)
         - This could be longer than the actual physical length of the link due to the orientation of axes
       * The $\underrightarrow{z}_i$ are the axes of each joint – axis of rotation for revolute joints, axis of translation for prismatic joints
       * The intersection of this line and the link axes are the reference points O_i
         - Note if $\underrightarrow{z}_i$ and $\underrightarrow{z}_{i-1}$ are parallel, this reference point can be anywhere

        - Note O_i is not fixed with respect to link i, but link $i-1$ instead (for a prismatic joint, O_i can shift)
    2. Link twist (α_i)
        * This is the angle between $\underrightarrow{z}_{i-1}$ and $\underrightarrow{z}_i$
    3. Link offset (d_i)
        * This is the distance along $\underrightarrow{z}_i$ from O_i to the intersection of $\underrightarrow{x}_i, \underrightarrow{z}_i$
        * This is a variable if the joint is prismatic, fixed if the joint is revolute
    4. Joint angle (θ_i)
        * This is the angle between $\underrightarrow{x}_i$ and $\underrightarrow{x}_{i-1}$
        * This is a variable if the joint is revolute, fixed if the joint is prismatic
- Note that this is referred to as the *modified* DH parameters

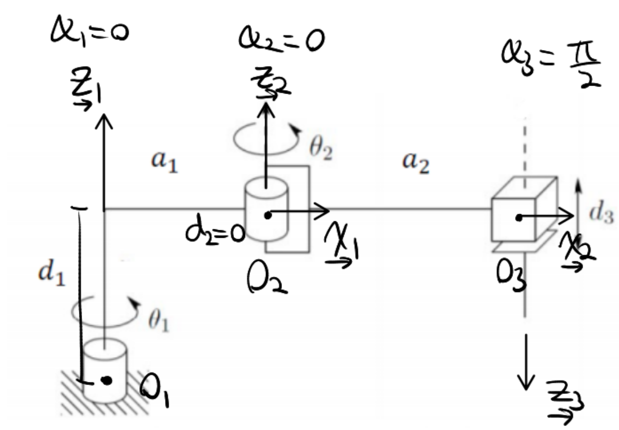

Figure 3: Example: SCARA manipulator DH parameters.

- The relative position of O_{i+1} from O_i is $\rho_i^{i+1} = d_i \underrightarrow{z}_i + a_i \underrightarrow{x}_i$
    - In $\underrightarrow{F}_i$ we have $\underrightarrow{\rho}_i^{i+1} = \begin{bmatrix} a_i \\ 0 \\ d_i \end{bmatrix}$
- Consider an arbitrary point P with position $\underrightarrow{v}_i$ relative to O_i; then $\underrightarrow{v}_i = \underrightarrow{v}_{i+1} + \underrightarrow{\rho}_i^{i+1} = \underrightarrow{v}_{i+1} + d_i \underrightarrow{z}_i + a_i \underrightarrow{x}_i$
    - The rotation matrix from $\underrightarrow{\mathcal{F}}_{i-1}$ to $\underrightarrow{\mathcal{F}}_i$ is $\boldsymbol{C}_{i,i-1} = \boldsymbol{C}_3(\theta_i)\boldsymbol{C}_1(\alpha_i)$ (first rotate about $\underrightarrow{x}_{i-1}$, then rotate about $\underrightarrow{z}_i$)
    - Therefore $\boldsymbol{T}_{i,i+1} = \begin{bmatrix} \boldsymbol{C}_{i,i+1} & d_i \mathbf{1}_3 + a_i \mathbf{1}_1 \\ \mathbf{0}^T & 1 \end{bmatrix}$
    - Expanded out: $\boldsymbol{T}_{i,i+1} = \begin{bmatrix} \cos(\theta_{i+1}) & -\sin(\theta_{i+1}) & 0 & a_i \\ \sin(\theta_{i+1})\cos(\alpha_{i+1}) & \cos(\theta_{i+1}\cos(\alpha_{i+1}) & -\sin(\alpha_{i+1}) & 0 \\ \sin(\theta_{i+1})\sin(\alpha_{i+1}) & \cos(\theta_{i+1})\sin(\alpha_{i+1}) & \cos(\alpha_{i+1}) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$